

Administrative

1. Review the information on the course webpage

<http://www.math.lsa.umich.edu/~jchw/2025Math592.html>

under **Course Information**. Please pay particular attention to the homework policy, and to the midterm and final exam dates. Contact Jenny (jchw@umich.edu) if you have any questions.

2. Please contact Jenny as soon as possible if you anticipate a conflict with a midterm or the final exam.
3. Complete the Math 592 Entrance survey (link) by 5pm on Friday 10 January.
4. Let Jenny know if there are any accommodations that would let you participate more fully in the course. For accommodations based on the impact of a disability that require modifications to class policies (such as extra time for exams), you will likely be asked to obtain documentation through the Office of Services for Students with Disabilities (SSD).

Assignment questions

Submit these questions through Gradescope by 5pm on Friday. You can find Gradescope submission instructions on the course webpage.

1. **(Functions review.)** Let $f : X \rightarrow Y$ be a function of sets X and Y . Recall that, for $A \subseteq X$, the *image* of A under f is the subset of Y

$$f(A) = \{f(a) \in Y \mid a \in A\} \subseteq Y.$$

For $C \subseteq Y$, the *preimage* of C under f is the subset of X

$$f^{-1}(C) = \{c \in X \mid f(c) \in C\} \subseteq X.$$

Let $A, B \subseteq X$ and $C, D \subseteq Y$. For each of the following, determine whether you can replace the symbol $\square$ with $\subseteq, \supseteq, =$, or none of the above. No justification necessary.

- (a) $f(A \cap B) \square f(A) \cap f(B)$ (b) $f(A \cup B) \square f(A) \cup f(B)$
- (c) For $A \subseteq B$, $f(B \setminus A) \square f(B) \setminus f(A)$
- (d) $f^{-1}(C \cup D) \square f^{-1}(C) \cup f^{-1}(D)$ (e) $f^{-1}(C \cap D) \square f^{-1}(C) \cap f^{-1}(D)$
- (f) For $C \subseteq D$, $f^{-1}(D \setminus C) \square f^{-1}(D) \setminus f^{-1}(C)$
- (g) $A \square f^{-1}(f(A))$ (h) $C \square f(f^{-1}(C))$

2. **(Cartesian product review.)** For sets X and Y , let $A, B \subseteq X$ and $C, D \subseteq Y$. Consider the Cartesian product $X \times Y$. For each of the following subsets, determine whether you can replace the symbol $\square$ with $\subseteq, \supseteq, =$, or none of the above. No justification necessary.

- (a) $(A \times C) \cup (B \times D) \square (A \cup B) \times (C \cup D)$
- (b) $(A \times C) \cap (B \times D) \square (A \cap B) \times (C \cap D)$
- (c) $(X \setminus A) \times (Y \setminus C) \square (X \times Y) \setminus (A \times C)$