

Name: \_\_\_\_\_

Score (Out of 6 points):

1. (a) (2 points) Prove that a space  $X$  is contractible if and only if the identity map  $id_X$  is homotopic to the constant map  $c_{x_0}$  at some fixed point  $x_0 \in X$ .

$$\begin{aligned} id_X : X &\longrightarrow X \\ x &\longmapsto x \end{aligned}$$

$$\begin{aligned} c_{x_0} : X &\longrightarrow X \\ x &\longmapsto x_0 \end{aligned}$$

- (b) (4 points) Suppose that a space  $X$  is contractible, that is, the identity map  $id_X$  is homotopic to the constant map  $c_{x_0}$  at some point  $x_0 \in X$ . Show that the identity map is homotopic to the constant map  $c_x : X \rightarrow X$  at **any** point  $x \in X$ .