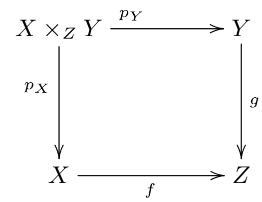
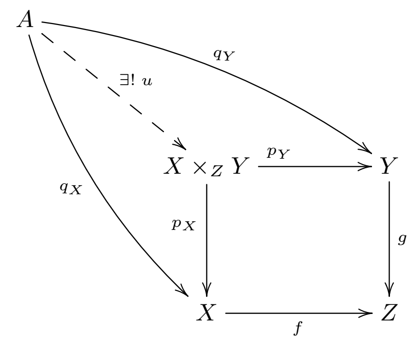
Name: ______________________________ Score (Out of 5 points):

Definition. Let $\mathcal{C}$ be a category with morphisms $f : X \to Z$ and $g : Y \to Z$. The *pullback* of f and g, if it is exists, is an object $X \times_Z Y$ of $\mathcal{C}$ along with morphisms $p_X : X \times_Z Y \to X$ and $p_Y : X \times_Z Y \to Y$ that make the following diagram commute,



and satisfy the following universal property. Whenever there is an object A and morphisms $q_X : A \to X$ and $q_Y : A \to Y$ satisfying $f \circ q_X = g \circ q_Y$, then there is a unique morphism $u : A \to X \times_Z Y$ as shown that makes the diagram commute.



1. (a) (1 point) Let $\mathcal{C}$ be a category, and let $f : X \to Z$ and $g : Y \to Z$ be two morphisms in $\mathcal{C}$. Give a precise statement of what it means to say that the universal property defines the pullback 'uniquely up to unique isomorphism'. (You do not need to prove this statement).

(b) (4 points) Let Top be the category of topological spaces and continuous maps, and let $f : X \to Z$ and $g : Y \to Z$ be two continuous maps. Show that the pullback $X \times_Z Y$ is the subspace of the product $X \times Y$ (with the product topology),

$$X \times_Z Y = \{(x, y) \mid f(x) = g(y)\} \subseteq X \times Y$$

and the maps $p_X : X \times_Z Y \to X$ and $p_Y : X \times_Z Y \to Y$ are the restrictions of the projection maps $X \times Y \to X$ and $X \times Y \to Y$, respectively.