Math 471 - Introduction to Numerical Methods - Fall 2019

Assignment # 10.
Due: Thursday, December 5, 2019.

1. Find the solution of the equation $y' = 2y$, $y(0) = 1$ by
   (a) Taylor's method with m=1. (Euler's method)
   (b) Runge-Kutta method of order 2. (Corrected Euler)
   (c) Adam-Bashforth method of order 4. (use exact solution to start the method).

   Use step size h=0.1, then repeat with h=0.05,0.001. In cases (1)-(3), perform the first 2 steps by hand (i.e. compute y_1 and y_2 by hand). Write a small program to find the solution at x=1. Find the exact solution, compute the error and error/h^p for the various step sizes, where p=1,2 and 4 in the respective cases.

2. In class, we have derived the error formula for Euler's method. For $y' = 2y$, $y(0) = 1$, use the error formula to find an upper bound for the error at x=1. How small do we need to take h to ensure an error of less than 10^{-4}?.

3. Perform one step of Richardson's extrapolation to get an improved solution at x=1, using values obtained with h=0.1 and h=0.05 by the second order Runge-Kutta method. Compare with exact solution.

4. Construst the quadratic interpolatory polynomial through $(x_{n-2}, f_{n-2}), (x_{n-1}, f_{n-1})$ and (x_n, f_n). $(f_k = f(y_k))$. Integrate this polynomial from x_n to x_{n+1} to obtain the third order Adam-Bashforth rule.

5. Solve $y' = 2y$, $y(0) = 1$ by the second order predictor-corrector method:
   (a) Predictor: $u^0_{n+1} = u_n + hf(u_n)$
   (b) Corrector: $u^{k+1}_{n+1} = u_n + \frac{h}{2}(f(u_n) + f(u^k_{n+1}))$

   Compute the solution at $x = 1$. Use h=0.2 and (a) one correction iteration per step (b) two correction iterations per step.

6. Consider the IVP for a nonlinear pendulum, $y''(t) + \sin(y(t)) = 0, y(0) = 1, y'(0) = 0$. Convert the problem into a first order system and use 4th order Runge-Kutta with h=0.5 to solve for y(1) and y'(1).