1. Consider the function \( f(x) = \ln(1 + x) \).
   a) Write down the Taylor polynomial of degree \( n \) about \( x_0 = 0 \).
   b) How large must \( n \) be in order to approximate \( \ln(1.1) \) to within \( 10^{-5} \)?

2. Consider the function \( f(x) = \sin(\pi x/2) \).
   a) Expand \( f(x) \) in a Taylor series about the point \( x_0 = 0 \).
   b) Find an expression for the remainder.
   c) Estimate the number of terms that would be required to guarantee accuracy of \( 10^{-6} \) for \( f(x) \) for all \( x \) in the interval \([-1, 1]\). Write explicitly the resulting Taylor polynomial.
   d) On the same graph, plot \( f(x) \) and its 1st, 3rd, 5th and 7th degree Taylor polynomials over \([-2, 2]\). Discuss what you observe. How accurate are the successive Taylor polynomials in approximating \( f(x) \)?

3. Let \( A \) be the \( 2 \times 2 \) matrix \( A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \). Use Gauss elimination to obtain \( A^{-1} \) by solving the two systems \( Ax_1 = e_1 \) and \( Ax_2 = e_2 \), where \( e_1 \) and \( e_2 \) are the columns of the \( 2 \times 2 \) identity matrix. Note that you can perform both at the same time by considering the augmented system \([A|I]\). Show that \( A^{-1} \) exists if and only if \( det(A) \neq 0 \).

4. Let 
   \[
   E_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -m_{2,1} & 1 & 0 & 0 \\ -m_{3,1} & 0 & 1 & 0 \\ -m_{4,1} & 0 & 0 & 1 \end{pmatrix}, \quad
   E_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -m_{3,2} & 1 & 0 \\ 0 & -m_{4,2} & 0 & 1 \end{pmatrix}, \quad
   P_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.
   
   (a) Show that 
   \[
   E_1^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ m_{2,1} & 1 & 0 & 0 \\ m_{3,1} & 0 & 1 & 0 \\ m_{4,1} & 0 & 0 & 1 \end{pmatrix},
   
   (b) Show that 
   \[
   E_1^{-1}E_2^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ m_{2,1} & 1 & 0 & 0 \\ m_{3,1} & m_{3,2} & 1 & 0 \\ m_{4,1} & m_{4,2} & 0 & 1 \end{pmatrix}.
   
   (c) Show that \( P_1^{-1} = P_1 \).
5. Find the $LU$ factorization of $A$ and use it to solve $Ax = b$.

$$A = \begin{pmatrix} 4 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 4 \end{pmatrix}, \ b = \begin{pmatrix} 3 \\ -2 \\ 2 \\ -3 \end{pmatrix}.$$

6. Gaussian Elimination: Operation count. Prove that $\sum_{k=1}^{n} k^2 = \frac{n(n + 1)(2n + 1)}{6}$