1. **Elementary Matrices**

   Let $B$ be a $4 \times 4$ matrix to which we apply the following operations.
   - Double column 1,
   - halve row 3,
   - add row 3 to row 1,
   - interchange columns 1 and 4,
   - subtract row 2 from each of the other rows,
   - replace column 4 by column 3,

   (a) Write the result as a product of seven matrices, including $B$.

   (b) Write the result again as a product of three matrices, including $B$.

2. **Special Matrices:**

   Consider the matrix
   \[
   \begin{pmatrix}
   b & -1 & 0 \\
   -1 & 4 & 1 \\
   0 & 1 & 5
   \end{pmatrix}
   \]

   (a) For what values of $b$ is this matrix positive definite?

   (b) For what values of $b$ is this matrix strictly diagonally dominant? Recall that an $n \times n$ matrix $A$ is said to be strictly diagonally dominant if
   \[
   \sum_{j=1, j \neq i}^{n} |a_{ij}| < |a_{ii}| \quad \text{for } i = 1, \ldots, n.
   \]

3. **Partial Pivoting**

   (a) Prove that the matrix
   \[
   \begin{bmatrix}
   0 & 1 \\
   1 & 1
   \end{bmatrix}
   \]
   does not have an $LU$ decomposition.

   (b) Does the system
   \[
   \begin{bmatrix}
   0 & 1 \\
   1 & 1
   \end{bmatrix}
   \begin{bmatrix}
   x \\
   y
   \end{bmatrix}
   =
   \begin{bmatrix}
   a \\
   b
   \end{bmatrix}
   \]
   have a unique solution for all $a, b \in \mathbb{R}$? How do you know?

   (c) How can you modify the system in part (b) so that $LU$ decomposition applies?
4. Consider the linear system \( Ax = b \) where \( A \) is the following matrix,

\[
A = \begin{pmatrix}
-5 & 2 & -1 \\
1 & 0 & 3 \\
3 & 1 & 6
\end{pmatrix}.
\]

(a) Using Gaussian Elimination with partial pivoting to reduce the matrix to upper triangular form. Show clearly each step. Write the corresponding \( L \), \( U \) matrices. Multiply the \( L \), \( U \) matrices, what do you get? Determine the permutation matrix \( P \) such that \( PA = LU \).

(b) Use the \( P \), \( L \), \( U \) decomposition found in (a) to find the solution to \( Ax = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \).

Be sure to show all relevant steps.

(c) Use the \( P \), \( L \), \( U \) decomposition found in (a) to find the solution to \( Ax = \begin{pmatrix} 0 \\ 1 \\ 5 \end{pmatrix} \).

Again, clearly show all relevant steps.

5. **Cholesky Factorization**

Find the Cholesky factorization \( A = LL^T \) by hand for

\[
A = \begin{pmatrix}
4 & 1 & 0 \\
1 & 4 & 1 \\
0 & 1 & 4
\end{pmatrix}.
\]

6. **Gaussian Elimination (with Partial Pivoting)**

Write a program to solve an \( n \times n \) linear system using Gaussian Elimination. As a test of your code, run your code on the system

\[
A = \begin{pmatrix}
2 & 2 & -3 \\
3 & 1 & -2 \\
6 & 8 & 1
\end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ -1 \end{pmatrix}.
\]

**Extra Credit:** Incorporate partial pivoting into your code, test it on the above system.

*Print and attach the text file containing your program. No code, no credit.*