Math 471 - Introduction to Numerical Methods - Fall 2019

Assignment # 7.
Due: Thursday, October 31, 2017.

1. Consider Hermite interpolation for $n = 1$, $x_0 = 0$, and $x_1 = 1$. Compute (by hand) $\tilde{h}_1(x)$ using the Lagrange polynomials and using the Newton form (from the divided difference table) and then plot $\tilde{h}_1(x)$.

2. The theorem describing the error in using Hermite interpolation is as follows.
Theorem: If $f \in \mathscr{C}^{2n+2}[a, b]$, then

$$f(x) = H(x) + \frac{(x - x_0)^2 \dots (x - x_n)^2}{(2n + 2)!} f^{(2n+2)}(\xi)$$

for some ξ with $a < \xi < b$.

Consider $f(x) = x \ln x$, $n = 1$, $x_0 = 1$, and $x_1 = 3$.

   (a) Use linear interpolation and Hermite interpolation to approximate the value of $f(1.5)$. Which estimate is more accurate?

   (b) Verify that the error bound for Hermite interpolation holds for the Hermite polynomial found in (a).

3. Find a polynomial of least degree satisfying:

$$p(1) = -1\ ,\ \ p'(1) = 2\ ,\ \ p''(1) = 0\ ,\ \ p(2) = 1\ ,\ \ p'(2) = -2$$

4. Find the natural cubic spline $S(x)$ satisfying

$$S(0) = 0, \qquad S(1/2) = 1, \qquad S(1) = 0\,.$$

Your answer will be 2 cubic polynomials, $S_0(x)$, $S_1(x)$. Verify that your answer satisfies all the necessary conditions (interpolation, continuity of 1st and 2nd derivatives, boundary conditions).

5. (a) In the case of the clamped spline, the column vector of unknowns is $m = (M_0, M_1, \dots, M_{n-1}, M_n)^T$. Note that the equations for M_0 and M_n are no longer $M_0 = 0$ and $M_n = 0$, so that the tridiagonal matrix B will change slightly. Write down the matrix and right hand side for the linear system $Bm = f$ which determines m. Show that the matrix B is invertible, and hence the clamped cubic spline exists and is unique. (Hint: Show that the matrix B is diagonally dominant, hence invertible.)

   (b) Determine the clamped cubic spline $S(x)$ that interpolates the data $f(0) = 0$, $f(1) = 1$, $f(2) = 2$ and satisfies $S'(0) = S'(2) = 1$. Again, your answer will consist of 2 cubic polynomials, $S_0(x)$, $S_1(x)$. Verify all the necessary conditions and note that the boundary conditions for the clamped spline are different from those for the natural spline. Plot the spline over the interval $[0, 2]$.

6. The following data describe the shape of a car called "Buggy". Points (x, y) and (v, w) describe the upper and lower surfaces of the car respectively.

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x=[0.0  0.5  1.0  1.5  1.7  1.85  2.0  2.5  3.0  3.5  4.0  4.5  5.0  5.5  5.75  6.0];
y=[0.0  0.9  1.2  1.35  1.4  1.7  1.95  2.3  2.35  2.4  2.35  2.25  1.8  1.0  0.7  0.0 ];

v=[0.0  0.5  1.0  1.25  1.5  1.75  2.0  2.25  2.5  2.75  3.0  3.25  3.5  3.75  4.0
   4.25  4.5  4.75  5.0  5.25  5.5  5.75  6.0];

w=[0.0  0.0  0.0  0.0  0.0  0.45  0.6  0.45  0.0  0.0  0.0  0.0  0.0  0.0  0.0
   0.45  0.6  0.45  0.0  0.0  0.0  0.0  0.0];
```

(a) Approximate the shape of the car using

i. polynomial interpolant;

ii. cubic spline interpolant with natural boundary conditions.

(You may use the built-in Matlab commands polyfit and spline).

(b) Plot the interpolatory polynomials. Which car would you rather drive?