Math 471 - Introduction to Numerical Methods - Fall 2019

Assignment # 8.
Due: Thursday, November 7, 2019.

1. From the following table of values of $f(x) = \sinh x$ find $f'(1.4)$ using

   (a) Backward differencing based on $f(1.3)$ and $f(1.4)$, and backward differencing based on $f(1.2)$ and $f(1.4)$. Perform one step of Richardson's extrapolation on these values.

   (b) Centered differencing based on $f(1.3)$ and $f(1.5)$, and centered differencing based on $f(1.2)$ and $f(1.6)$. Perform one step of Richardson's extrapolation on these values.

   (c) The 2^{nd} order 1-sided formula based on $f(1.2)$, $f(1.3)$ and $f(1.4)$.

   (d) Find $f''(1.4)$ to the highest possible degree of accuracy.

   Compare your results with the results $f'(1.4) = \cosh 1.4 = 2.1509$, and $f''(1.4) = \sinh 1.4 = 1.9043$.

x	1.2	1.3	1.4	1.5	1.6
$f(x)$	1.5095	1.6984	1.9043	2.1293	2.3756

2. Given $f(x_0), \quad f(x_0 \pm h)$

   (a) Use the interpolating polynomial at $\{x_0,\ x_0 + h\}$ to approximate $f'(x_0)$.

   (b) Use the interpolating polynomial at $\{x_0 - h,\ x_0,\ x_0 + h\}$ to approximate $f'(x_0)$ and $f''(x_0)$.

3. Apply the following methods to compute $I = \int_0^1 f(x)dx$: Midpoint rule, trapezoid rule over [0,1], trapezoid rule first over [0,0.5] and then over [0.5,1], Simpson's rule over [0,1] for

   (i) $f(x) = x \sin x$

   (ii) $f(x) = \begin{cases} x & 0 \le x \le 0.5 \\ 1 - x & 0.5 \le x \le 1.0 \end{cases}$

   Compare your results with the exact answers.

4. Use the finite difference method to solve the 2-point Boundary Value Problems:

   (a) $\epsilon y'' - y = -1$ for $0 < x < 1,\ y(0) = -1,\ y(1) = 1$
   (b) $\epsilon y'' - y = -2x$ for $0 < x < 1,\ y(0) = -1,\ y(1) = 1$

   Use $\epsilon = 0.001$, and step sizes $h = 1/n$, $n = 4, 8, 16, 32, 64, 128$. Let y_h be the exact solution evaluated at the mesh points and and u_h the computed solution. Plot the exact and numerical solutions so that they are easy to compare (if you are using Matlab, try to use the subplot command to save paper). Set up a table containing the values: h, $||y_h - u_h||_\infty$, $||y_h - u_h||_\infty/h^2$. Comment on the order of accuracy of the numerical solution.