Math 471 Midterm List of Topics

- **Lecture notes**:
  - **Preliminaries:**
    Number systems, bases conversions,
    Floating point representation, errors (rounding, chopping, significant digits)
    Error propagation (floating point arithmetic, subtractive cancellation, function evaluation and condition)
    Big Oh notation
    Taylor's theorem, error estimation.
  - **Numerical Solutions of Linear systems:**
    Linear algebra preliminaries (existence and uniqueness of solution, invertibility, determinants, eigenvalues)

    Direct methods:
    Gaussian elimination, operation count, tridiagonal/banded systems.
    Gaussian elimination in matrix form, elementary matrices, LU factorization
    Symmetric positive matrices: Choleski factorization $A = LL^T$, also $A = LDL^T$
    Gaussian elimination with multiples RHSs
    Partial pivoting
    Vector norms, matrix norms, matrix condition number and errors

    Iterative methods: Jacobi, Gauss-Seidel, SOR.
    Iteration matrix, spectral radius and convergence
    Optimal SOR
  - **Rootfinding:**
    Preliminaries (Mean Value Theorem and existence of a root
    Bisection, Secant method, Newtons method
    Convergence and error estimates
    Fixed point iteration, existence/uniqueness of root,
    Order of convergence (linear, quadratic etc)
    Multiple roots
    Polynomial rootfinding
    Newtons method for nonlinear systems
- **Homework assignments 1-5**.

- **From the Textbook** (Brady, A friendly introduction to Numerical Analysis):
  - Chapter 1 (1.1-1.4): convergence, floating point representation, floating point arithmetic
  - Chapter 2 (2.1, 2.3-2.7): rootfinding methods (bisection, fixed point iteration, Newton's, secant, accelerated convergence methods), multiple roots, roots of polynomials.
  - Chapter 3 (3.0-3.5, 3.7, 3.8, 3.10): direct methods for solving a system of linear equations (Gaussian elimination, LU decomposition, Cholesky decomposition, pivoting, vector and matrix norms, error analysis), iterative methods (Jacobi, Gauss-Seidel, SOR), nonlinear systems of equations.
  - Appendix A: Important theorems from calculus
  - Appendix B: Algorithm for solving a tridiagonal system of linear equations