M555 - Introduction to Complex Variables - Winter 2009

Assignment # 6. Due: Thursday, February 19, 2009.

From the Textbook:

p.163 – Problem 7.
p.172 – Problem 2.
p.181 – Problem 4.
p.189 – Problems 2, 9, 11b

Additional Problems (Extra Credit):

- A Find the maximum modulus of $f(z) = z^2 + 1$ on the closed unit disk $\{z : |z| \le 1\}$.
- B Consider the power series

$$1 + z + z^2 + z^3 + z^4 + \cdots$$

What is the radius of convergence of this power series? Show that this power series fails to converge for any point on the unit circle |z| = 1.

C Consider the power series

$$1 + z^2 + z^4 + z^8 + z^{16} + \cdots$$

What is the radius of convergence of this power series? Given any point a on the unit circle |z| = 1, show that there are points on the unit circle arbitrarily close to a at which this power series diverges to ∞ .

"For points arbitrarily close to a on the unit circle" means that given any $\epsilon > 0$, there exists a point z with |z| = 1 and $|z - a| < \epsilon$ for which the property holds.