

MATH 572 - Numerical Methods for Scientific Computing II - 2015

Assignment # 1.

Due: Thursday, January 22, 2015.

1. Observed Order of Accuracy

I. Consider $u(x) = e^x$, $x_0 = 0$. Compute $D_+u(x_0)$, $D_0u(x_0)$, $D_3u(x_0)$, using $h = 0.1, 0.05, 0.025, 0.0125$. Compute the error. For each finite difference approximation, construct a table of: h , error, error/ h , error/ h^2 and error/ h^3 , confirm the expected order of accuracy. Compute the constants C and compare with theoretical values. Plot $\log(\text{error})$ against $\log h$.

II. Perform Richardson's extrapolation on the values of $D_+u(x_0)$, and $D_0u(x_0)$.

2. Roundoff Errors

Consider again $u(x) = e^x$, $x_0 = 0$. Compute $D_+u(x_0)$ for a sequence of increasingly small h values, for example, $h = 10^{-n}$. Plot the error against h and explain your observations.

3. Finite Differences using interpolating polynomials

Let $p(x)$ be the interpolating polynomial to $u(x)$ at the points x_0 , $x_0 - h$ and $x_0 - 2h$. Show that $p'(x_0)$ gives exactly the 3-point one-sided difference approximation $D_2(x_0)$ derived in class.

4. High order approximations using Richardson's extrapolation

Consider the backward difference approximation $D_-u(x_0)$ based on a step size h , and on a step size $2h$. Show that Richardson's extrapolation on the two approximations gives $D_2(x_0)$.

5. Finite difference approximations with unequally spaced points.

I. Use the method of undetermined coefficients to derive the 3-point difference approximation to $u''(x_0)$ using the unequally spaced points $u(x_0 - h_1)$, $u(x_0)$ and $u(x_0 + h_2)$.

$$u''(x_0) \approx \frac{2u(x_0 - h_1)}{h_1(h_1 + h_2)} - \frac{2u(x_0)}{h_1h_2} + \frac{2u(x_0 + h_2)}{h_2(h_1 + h_2)}$$

Comment on the accuracy of the approximation.

II. Use this formula to approximate the derivative of e^x at $x_0 = 0$ when

- $h_1 = 0.05$ and $h_2 = 0.1$
- $h_1 = 0.005$ and $h_2 = 0.01$
- $h_1 = 0.0005$ and $h_2 = 0.001$

Compare with the exact answer. By what factor is the error decreasing at each step?

III. Confirm that the approximation reduces to the centered formula when $h_1 = h_2 = h$ and use it to compute the second derivative with (a) $h = 0.1$ (b) $h = 0.01$ and (c) $h = 0.001$. By what factor is the error decreasing at each step?

IV. Discuss the accuracy of centered vs. non-centered approximations.