

M572 - Numerical Methods for Scientific Computing II - 2015

Assignment # 6.

Due: Thursday, March 12, 2015.

1. The trapezoid method is $u^{n+1} = u^n + \frac{k}{2}(f(u^n) + f(u^{n+1}))$.

I. Show that the LTE is $O(k^2)$.

II. Show that the method is A-stable.

III. Show directly that the method converges (i.e. that $\lim_{k \rightarrow 0} e^n = 0$. Hint:

Obtain a relationship between e^{n+1} and e^n and proceed as in the convergence proof for Euler's method).

2. Solve the following difference equations.

I. $u^{n+2} - 2u^{n+1} - 3u^n = 0$, $u^0 = 0$, $u^1 = 1$.

II. $u^{n+3} - 3u^{n+2} + 3u^{n+1} - u^n = 0$, $u^0 = 1$, $u^1 = 0$, $u^2 = -3$.

3. Consider Milne's method

$$u^{n+1} = u^{n-1} + \frac{k}{3}(f(u^{n+1}) + 4f(u^n) + f(u^{n-1}))$$

I. Show that the LTE is $O(k^4)$.

II. Show that the root condition is satisfied.

III. Show that when the method is applied to $u' = \lambda u$, the roots of the characteristic polynomial satisfy

$$\zeta_1 = e^{\lambda k} + O(k^5), \quad \zeta_2 = -e^{-\frac{\lambda k}{3}} + O(k^3).$$

IV. Compute the solution of $u' = u$, $u(0) = 1$, using $k = 0.1$. Take $u^0 = u(0)$ and use the forward Euler method to compute the starting value u^1 . Compute the error at $t = 5$. Repeat with $k = 0.01$. Now apply the method to $u' = -u$, $u(0) = 1$. Explain the results.

4. Derive the 3-step Backward Differentiation Method.

5. Consider the initial value problem

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}' = \begin{pmatrix} -11 & 9 \\ 9 & -11 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_0 = \begin{pmatrix} 1.0 \\ 1.2 \end{pmatrix}$$

Find the exact solution. Compute the solution for $0 \leq t \leq 2$ using the forward Euler and the backward Euler methods with step size $k = 0.15, 0.12, 0.11, 0.10, 0.09$ and 0.05 . Plot the first component of the solution vs. time (use the MATLAB command `subplot(221)`). Explain the results.

6. Use the boundary locus method to find the absolute stability region of (i) the 2-stage RK and (ii) the 2-step BDF method.