Math 493 Fall 2023 HW7

- 0. (10 bonus points) Work on at least part of this homework with two other students from this class that you have not worked with previously. Describe which problems you worked on together. Also, include one fact about each of these students that you did not know about them.
- 1. (30 points) Classify all plane crystallographic groups with point group isomorphic to D_3 .
- 2. (20 points) Write down the character tables for D_4 and Q (the quaternion group).
- 3. (30 points) Classify the irreducible representations of the dihedral group D_n up to isomorphism. (You will want to consider separately the cases of n even and n odd. Also recall that in HW3, we found the conjugacy classes in D_n , which will be helpful in solving this problem. You may use the fact that the number of irreducible representations of a finite group G equals the number of conjugacy classes in G, in order to show that you have exhausted all the irreps.)
- 4. (10 points) Can you use the character table of a finite group G to determine if G is simple or not?
- 5. (10 points) Let k be a field of characteristic p. Let $G = C_p$ act on the two dimensional k-vector space $V = k^2$ by sending a generator of G to the matrix

$$\begin{bmatrix} 1 & 1 \\ & 1 \end{bmatrix} \in \operatorname{GL}_2(k),$$

which has order p. Show that this representation is not a direct sum of irreducible representations.