

Math 614 Quiz 5

TRUE OR FALSE

- (1) The subring $\mathbb{C}[u^2, uv, v^2]$ of $\mathbb{C}[u, v]$ is **normal**.
- (2) The quotient ring $\mathbb{C}[x, y, z]/\langle xz - y^2 \rangle$ is a UFD.
- (3) The ring extension $\mathbb{R}[x(1-x), y, xy] \subset \mathbb{R}[x, y]$ is integral.
- (4) The ring extension $\overline{\mathbb{F}_7}[x, y] \subset \overline{\mathbb{F}_7}[x, y, z]/\langle zx - 1 \rangle$ is integral.
- (5) For the ring inclusion $\mathbb{F}_2[x, xy] \subset \mathbb{F}_2[x, y]$, every maximal ideal of $\mathbb{F}_2[x, y]$ contracts to a maximal ideal of $\mathbb{F}_2[x, xy]$.
- (6) The subring $\mathbb{Q}[x^2, x^3, y^2, y^5]$ of $\mathbb{Q}[x, y]$ has dimension two.
- (7) For the ring extension $\mathbb{R}[x, y] \subset \mathbb{R}[x, y, \sqrt{x^2 + y^5}, \sqrt[3]{xy}]$, the contraction map preserves the heights of prime ideals.
- (8) If a field extension $K \subset L$ has finite degree, then L is finitely generated as a K -algebra.
- (9) If a field L is finitely generated as a K -algebra (where K is a field), then L has finite degree over K .
- (10) The classes of the elements x^2, y, z are algebraically independent elements over \mathbb{Z} in the \mathbb{Z} -algebra $\mathbb{Z}[x, y, z, w]/\langle z^3 + xyz^2 + xyw, w - 1 \rangle$