Math 156 Applied Honors Calculus II Computer Lab Worksheet Fall 2019

Log in to your umich account, open Maple (from the dock or applications folder), then click on **File**, then select **New**, then click on **Worksheet Mode**. Type each command below and hit return to get the result. You may use the arrow keys or mouse to maneuver.

**arithmetic**

> 2+2
> 2^6
> sqrt(64)
> a:=3 *Be sure to include the colon before the equal sign.*
> a^2
> Pi
> evalf(Pi) *evalf = evaluate*

You can put several commands on a single line.

> sin(Pi); cos(Pi); tan(Pi)
> erf(0); erf(1); evalf(erf(1)) $\text{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$ : *error function (see hw3)*
> abs(-1)
> I; I^2 $\text{I} = \sqrt{-1}$, *an imaginary number, written "i" in math books*
> exp(1); evalf(exp(1))
> exp(Pi*I) *that's right,* $e^{\pi i} = -1$; *you'll learn why in December*

**limits**

> with(student) *This loads the student commands.*
> Limit(1/x,x=infinity) *Use the right arrow key after* 1/x.
> value(%) *"%" refers to the expression on the previous line.*
> limit(1/x,x=infinity) Limit *displays the limit,* limit *evaluates it.*
> limit(1/x,x=0)
> limit(1/x,x=0,left); limit(1/x,x=0,right) *This gives the one-sided limits.*
> limit(x*exp(-x),x=infinity) *Maple knows l'Hopital's rule.*

**plotting**

At the top of the screen, click **Maple 2018**, then **Preferences**, then **Display**, then in the **Plot display** menu, change **Inline** to **Window**, and click **Apply to Session**. This opens each plot in a separate window.

> plot(1/x,x=0..5,y=0..5) *After viewing, close the window to avoid clutter.*
> plot([1/x,1/x^2],x=0..5,y=0..5) *Which curve is* $1/x$? ... $1/x^2$?
> plot(tan(x),x=-2*Pi..2*Pi,y=-4..4) $\tan(x)$ *has vertical asymptotes at* $x = \pm\pi/2, \ldots$
> limit(tan(x),x=Pi/2)
> limit(tan(x),x=Pi/2,left); limit(tan(x),x=Pi/2,right)
> plot(arctan(x), x=-2*Pi..2*Pi,y=-4..4) $\arctan(x)$ *has horizontal asymptotes as* $x \to \pm\infty$
> limit(arctan(x),x=infinity)
> plot([exp(-x),exp(-x^2)],x=0..3,y=0..1) *Which curve is* $e^{-x}$? ... $e^{-x^2}$?

The next plot is an example of a parametric curve using polar coordinates.

> plot([sin(4*t),t,t=0..2*Pi],coords=polar) *Try changing 4 to 7 (for example).*

Maple has a help facility, e.g. the next command opens a window with help about plotting.

> ?plot

**Riemann sums**

> rightbox(x^2,x=0..1,2) *This plots the right-hand Riemann sum for* $\int_0^1 x^2 dx$ *with* $n = 2$.

> rightsum(x^2,x=0..1,2); evalf(%) *This evaluates the Riemann sum.* $\Delta x = ?$, $x_i = ?$

Repeat for $n = 4, 8, 16$ (use the arrow keys or mouse to maneuver). Do the results converge to the correct value as $n$ increases? The commands for the left-hand and midpoint Riemann sums are **leftbox**, **leftsum**, **middlebox**, **middlesum**. Repeat the previous commands, substituting *left* and *middle* in place of *right*. For a given value of $n$, which sum is the most accurate?

**antiderivatives**

> Int(x^n,x); value(%)
> int(x^n,x) Int *displays the integral,* int *evaluates it.*
> int(ln(x),x); diff(%,x)
> int(1/(x^2+1),x); diff(%,x)
> int(1/sqrt(x^2+1),x); diff(%,x) *We'll discuss* sinh(x) *later in the semester.*
> int(exp(-x^2),x); diff(%,x) *recall* : $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$

> f:=x^3/sqrt(1-x^8); int(f,x) *We can also substitute to find the antiderivative, as follows.*
> a:=Int(f,x)
> b:=changevar(x^4=u,a) $u = x^4,\ du = 4x^3 dx$
> c:=value(b)
> d:=subs(u=x^4,c); diff(d,x) *This returns to the original variable and checks the answer.*

**definite integrals**

> Int(x,x=a..b); value(%) *Oops! - we need to clear variables a and b.*
> a:='a'; b:='b'; Int(x,x=a..b); value(%) <u>ok</u>
> Int(sqrt(1-x^2),x=-1..1); value(%) *This gives the area of a semi-circle with radius 1.*
> Int(1/x^2,x=1..infinity); value(%)
> Int(1/x^2,x=-1..1); value(%) *Does the answer make sense?*
> Int(1/x,x=1..infinity); value(%)
> Int(exp(-x),x=0..infinity); value(%)
> Int(x*exp(-x),x=0..infinity); value(%) *Maple knows integration by parts.*
> Int(exp(-x^2),x=0..infinity); value(%) *This requires multivariable calculus (Math 215).*

**surfaces in 3D** (try these for fun, use the mouse to rotate the surface)

> with(plots)
> sphereplot(1,theta=0..2*Pi,phi=0..Pi) *sphere,* theta *is longitude,* phi *is latitude*
> plot3d(x^2-y^2,x=-1..1,y=-1..1) *saddle*
> plot3d([r*cos(theta),r*sin(theta),r],r=0..1,theta=0..2*Pi) *cone*
> plot3d([(1+.2*cos(a))*cos(b),.2*sin(a),(1+.2*cos(a))*sin(b)],a=0..2*Pi,b=0..2*Pi) *torus*

**Homework Assignment** (hand in with hw4 on Tuesday Oct 1)

In class and on hw we computed Riemann sums for the integral $I = \int_0^1 f(x)\,dx$ with $f(x) = e^x, e^{-x}$, and found that if $\Delta x$ decreases by a factor of 1/2, then the error in the right-hand Riemann sum $R_n$ decreases by about 1/2 and the error in the midpoint Riemann sum $M_n$ decreases by about 1/4. Is the same true for $f(x) = \sqrt{x}$? To answer this question, construct a table with the following data (you may use Maple or a calculator). column 1: $n$ (take $n = 2, 4, 8, 16$), column 2: $\Delta x$, column 3: $R_n$, column 4: $|I - R_n|$, column 5: $M_n$, column 6: $|I - M_n|$. For a given value of $n$, which method gives a more accurate answer? How do the results for $\sqrt{x}$ compare with the results for $e^x$, $e^{-x}$? What is similar? ... different? Explain your observations.