Math 156 Applied Honors Calculus II Fall 2023

hw1 , due: Tuesday, September 5 at 4pm

Write the solutions neatly; number each page; scan them using a scanning app; don't take a photograph (the background comes out too dark and the writing is hard to read); upload as a PDF into Gradescope through your section Canvas site.

0. (optional) Describe your academic background and interests. What year are you in? Have you decided on your major? How did you hear about this class? What other STEM classes are you taking this semester?

1. Write the sum in sigma notation.

a) $1+3+5+7+\cdots+(2n-1)$

b) $1+2+4+8+16+32$

c) $1+\dfrac{1}{4}+\dfrac{1}{9}+\dfrac{1}{16}+\dfrac{1}{25}+\dfrac{1}{36}$

d) $x+x^2+x^3+\cdots+x^n$

e) $1-x+x^2-x^3+\cdots+(-1)^n x^n$

2. True or False? Give a reason or counterexample to justify your answer.

a) $(f(x)g(x))' = f'(x)g'(x)$ b) $\displaystyle\sum_{i=0}^{n}(n-i)=\sum_{i=0}^{n} i$ c) $\displaystyle\sum_{i=1}^{5}((i+1)^3-i^3)=215$ d) $\dfrac{1}{2}+\dfrac{1}{3}=\dfrac{2}{5}$

3. Prove the following results. Follow the examples in class.

a) $\displaystyle\sum_{i=1}^{n}(a_i+b_i)=\sum_{i=1}^{n}a_i+\sum_{i=1}^{n}b_i$

b) $\displaystyle\sum_{i=1}^{n} i^2=\frac{n(n+1)(2n+1)}{6}$ (hint: start from $(i+1)^3-i^3=\ldots,$)

4. Each set below defines a region in the xy-plane. Sketch the region, express the area as a limit of Riemann sums, and evaluate the limit.

a) $\{(x,y): 0\le x\le 1,\ 0\le y\le 1+x\}$

b) $\{(x,y): -1\le x\le 1,\ 0\le y\le 1-x^2\}$

c) $\{(x,y): 0\le x\le 1,\ 0\le y\le e^x\}$

note: example (c) requires the formula for the sum of a finite geometric series; if you know it, then go ahead; otherwise skip it for now; you will learn it on hw2

5. The expression $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)^2\frac{2}{n}$ is a limit of Riemann sums giving the area of a region in the xy-plane defined by $\{(x,y): a\le x\le b,\ 0\le y\le f(x)\}$. Assuming $a=1$, sketch the region.

optional :

(i) There are two ways to evaluate the limit; do you know what they are? If so, explain.

(ii) Is there a different choice of a, b and $f(x)$ that gives a different region but the same area?

6. Sketch the graph of the function on the interval $0\le x\le 2\pi$. Label the axes.

a) $f(x)=\sin x$ b) $f(x)=\sin 2x$ c) $f(x)=\sin^2 x$