Math 156 Applied Honors Calculus II Fall 2024

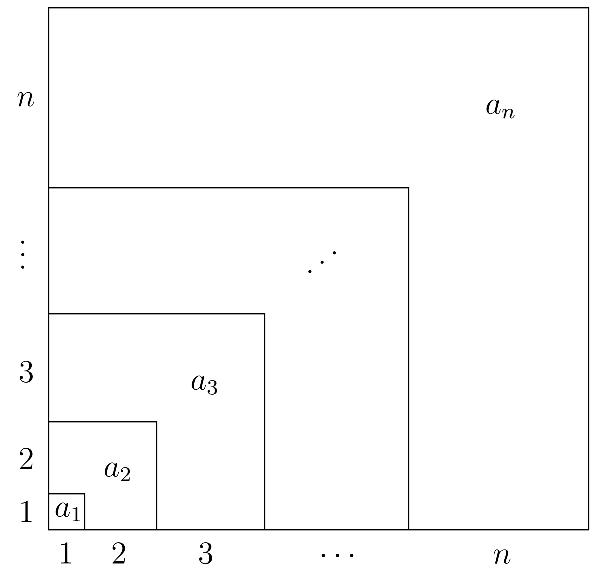
hw2 , due: Tuesday, September 10 at 4pm

1. Show that $\int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$ using Riemann sums.

2. Show that $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$ using two methods.

<u>method 1</u> Use a telescoping sum as in class.

<u>method 2</u> Consider a square where each side has segments of length $1, 2, \ldots, n$; then the side length is $1+2+\cdots+n = \frac{n(n+1)}{2}$ and the area of the square is $A = \left(\frac{n(n+1)}{2}\right)^2$. Now consider areas $a_1, a_2, \ldots, a_n$, where a_1 is a unit square and $a_2, a_3, \ldots, a_n$ are as shown in the figure. Show that $a_i = i^3$ for $i = 1, \ldots, n$. Hence the area of the square is also equal to $A = a_1 + a_2 + \cdots + a_n = \sum_{i=1}^{n} a_i = \sum_{i=1}^{n} i^3$.



3. Sketch the region $\{(x,y) : 0 \le x \le 1, 0 \le y \le x^3\}$ and find the area by (a) Riemann sums, (b) FTC.

4. Energy use (in watt-hours) is the integral of power (in watts) over time (in hours). The power consumed by an air conditioner is 600 watts at 4pm, 1000 watts at 6pm, and 400 watts at 8pm. Estimate the number of watt-hours used between 3pm and 9pm by the midpoint Riemann sum.

5. A steel rod of length L cm has variable cross-section area $A(x)$ cm^2, where x is measured in centimeters from one end of the rod. The rod has uniform mass density ρ g/cm^3. (a) Using Riemann sums, derive an integral for the total mass of the rod M. (hint: divide the rod into n slices, find the mass of each slice, ...) (b) Compute the total mass for the case $L = 25$ cm, $A(x) = (1+\sqrt{x})$ cm^2, $\rho = 8$ g/cm^3. Express M in kilograms (kg).

6. a) Derive the formula for the sum of a <u>finite geometric series</u>,

$$\sum_{i=0}^{n} r^i = 1 + r + r^2 + \cdots + r^n = \frac{1-r^{n+1}}{1-r}\ , \text{ if } r \ne 1.$$

(hint: first check the formula for $n = 0, 1, 2$, then show it's true for general n.)

b) A student obtains a \$1000 loan and repays 1/2 of the balance each year, i.e. \$500 is repaid in year 1, \$250 is repaid in year 2, and so on. Express the total amount repaid after 10 years as a finite geometric series (i.e. find the r and n) and evaluate it using the formula in part (a).

c) Evaluate $\int_0^1 e^x dx$ by Riemann sums. (this completes problem 4c from hw1)

d) What happens to the formula in part (a) in the limit $r \to 1$?

7. Let $I = \int_0^1 e^{-x} dx = 1 - e^{-1} = 0.63212056$, and let R_n, M_n be the right-hand and midpoint Riemann sums with n intervals. Construct a table as follows (use a calculator, keep 4 decimal digits, see page 6 in the notes). column 1: n (take $n = 1, 2, 4$); column 2: Δx; column 3: R_n; column 4: $|I - R_n|$; column 5: M_n; column 6: $|I - M_n|$. For a given value of n, which method is more accurate? When Δx decreases by a factor of 1/2, by what factor does the error approximately decrease for each method?

8. Sketch the graph of the function on the interval $0 \le x \le 2\pi$. Label the axes.

a) $f(x) = \cos x$, b) $f(x) = \cos 2x$, c) $f(x) = \cos^2 x$

<u>announcement</u> The Science Learning Center offers study groups for Math 156 students. Participation is voluntary, but many students find the groups helpful. Online registration starts on Thurs Sept 5 at 9am. If the group you want is full, join the waitlist and another group may be opened. `http:/www.lsa.umich.edu/slc/study-groups/join.html`