Math 156 Applied Honors Calculus II Fall 2023

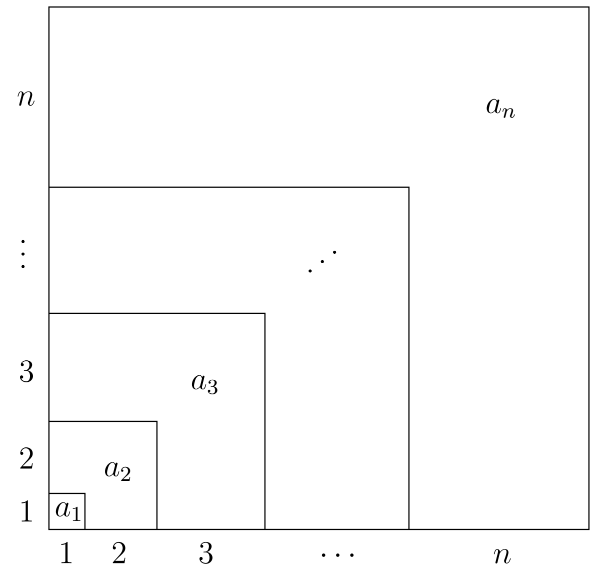
hw2 , due: Tuesday, September 12

1. Show that $\int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$ using Riemann sums.

2. Show that $\sum\limits_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$ using two methods.

<u>method 1</u> Use a telescoping sum as in class.

<u>method 2</u> Consider a square where each side has segments of length $1, 2, \ldots, n$, so that the side length is $1+2+\cdots+n = \frac{n(n+1)}{2}$ and the area of the square is $A = \left(\frac{n(n+1)}{2}\right)^2$. Now consider areas $a_1, a_2, \ldots, a_n$, where a_1 is a unit square and $a_2, a_3, \ldots, a_n$ are the areas shown in the figure. Show that $a_i = i^3$ for $i = 1, \ldots, n$. Hence the area of the square is also equal to $A = a_1 + a_2 + \cdots + a_n = \sum\limits_{i=1}^{n} a_i = \sum\limits_{i=1}^{n} i^3$.



3. Sketch the region $\{(x,y) : 0 \le x \le 1, 0 \le y \le x^3\}$ and find the area by (a) Riemann sums, (b) FTC.

4. a) Express $\lim\limits_{n\to\infty} \dfrac{1}{n} \sum\limits_{i=1}^{n} \dfrac{1}{1+(i/n)^2}$ as an integral. b) Find the derivative of $f(x) = \displaystyle\int_0^{x^2} \sqrt{1+t^3}\, dt$.

5. A steel rod of length L cm has variable cross-section area $A(x)$ cm^2, where x is measured in centimeters from one end of the rod. The rod has uniform mass density ρ g/cm^3. (a) Using Riemann sums, derive an integral for the total mass of the rod M. (hint: divide the rod into n slices, find the mass of each slice, ...) (b) Compute the total mass for the case $L = 25$ cm, $A(x) = (1+\sqrt{x})$ cm^2, $\rho = 8$ g/cm^3. Express M in kilograms (kg).

6. a) Derive the formula for the sum of a <u>finite geometric series</u>,

$$\sum_{i=0}^{n} r^i = 1 + r + r^2 + \cdots + r^n = \frac{1-r^{n+1}}{1-r} \ , \text{ if } r \ne 1.$$

(hint: first check the formula for $n = 0, 1, 2$, then show it's true for general n.)

b) A student obtains a \$1000 loan and repays 1/2 of the balance each year, i.e. \$500 is repaid in year 1, \$250 is repaid in year 2, and so on. Express the total amount repaid after 10 years as a finite geometric series (i.e. find the r and n) and evaluate it using the formula in part (a).

c) Evaluate $\int_0^1 e^x dx$ by Riemann sums. (this completes problem 4c from hw1)

d) What happens to the formula in part (a) in the limit $r \to 1$?

7. Let $I = \int_0^1 e^{-x} dx = 1 - e^{-1} = 0.63212056$, and let R_n, M_n be the right-hand and midpoint Riemann sums with n intervals. Construct a table as follows (use a calculator). column 1: n (take $n = 1, 2, 4$); column 2: Δx; column 3: R_n; column 4: $|I - R_n|$; column 5: M_n; column 6: $|I - M_n|$. For a given value of n, which method is more accurate? When Δx decreases by a factor of 1/2, by what factor does the error approximately decrease for each method?

8. Sketch the graph of the function on the interval $0 \le x \le 2\pi$. Label the axes.

a) $f(x) = \cos x$, b) $f(x) = \cos 2x$, c) $f(x) = \cos^2 x$

<u>announcement</u> The Science Learning Center offers study groups for Math 156 students. Participation is voluntary, but many students find the groups helpful. Online registration starts on Thurs Sept 7 at 9am. If the group you want is full, join the waitlist and another group may be opened. `http:/www.lsa.umich.edu/slc/study-groups/join.html`