Math 156 Applied Honors Calculus II Fall 2024

hw3 , due: Tuesday, September 17 at 4pm

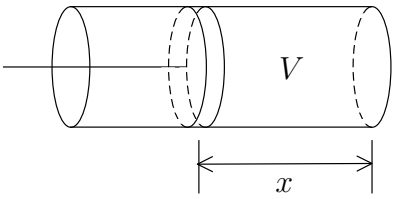
Write neatly, make sure the scan is clear, explain the steps.

1. Find the work done in raising a 60 kg mass to a height of 2 m above the floor.

2. A spring has natural length 20 cm and a 25 N force is needed to stretch it to length 30 cm. Find the work done in stretching the spring from length 20 cm to 25 cm.

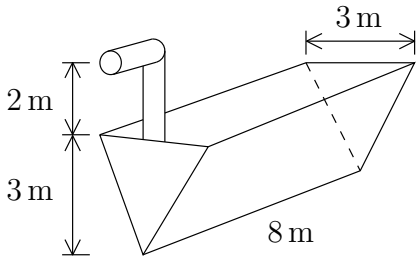
3. A flexible cable 50 ft long weighing 0.5 lb/ft hangs from the top of a building 120 ft high. a) Find the work done in pulling the cable to the top of the building. b) Find the work done in pulling half the cable to the top of the building. Express the answers in ft-lb. (hint for (a) and (b): draw a picture and an axis)

4. a) A volume of compressed gas in a closed cylinder expands as a piston is withdrawn from the cylinder. The gas pressure is a function of the gas volume, $P = P(V)$. The force exerted by the gas on the piston is the product of the piston surface area and the gas pressure, $f = \pi R^2 P$, where R is the cylinder radius. Show that the work done when the gas expands from volume V_1 to V_2 is $W = \int_{V_1}^{V_2} P(V)dV$. (hint: start from $W = \int_a^b f(x)dx$, where x is the piston displacement as shown in the figure and $f(x)$ is the force exerted by the gas on the piston, then change variables from x to V)



b) In a steam engine the steam pressure P and volume V satisfy $PV^{1.4} = k$, where k is a positive constant. Note that P decreases when V increases. Use part (a) to find the work done by the engine when the steam starts at pressure 1600 lb/in^2 and volume 100 in^3 and expands to volume 800 in^3. Show the steps in computing the integral. Express the answer in ft-lb.

5. A tank with the indicated shape is full of water. Find the work done in pumping the water to the top of the outlet. Use ρ = 1000 kg/m^3 for the water density. Express the final result in MJ. (hint: follow the steps in the example from class; you may assume no work is required to move the water horizontally)



6. The error function, defined by $\text{erf}(x) = \dfrac{2}{\sqrt{\pi}} \displaystyle\int_0^x e^{-t^2} dt$, is used in physics and probability.

Find the following quantities; compute (c) using the midpoint Riemann sum with $n = 2$ intervals.

a) $\text{erf}(0)$ b) $\text{erf}'(0)$ c) $\text{erf}(1)$ d) $\lim_{x\to\infty} \text{erf}'(x)$ e) $\lim_{x\to\infty} \text{erf}(x)$

f) Sketch the graph of $\text{erf}(x)$ for $x \geq 0$ using the results from (a)-(e).

hint for e): use the fact that $\displaystyle\int_0^\infty e^{-x^2} dx = \sqrt{\pi}/2$

7. A company buys an oil tank and fills it with g_0 gallons of oil. Each year a portion of the oil is consumed by operations and a new shipment of oil is added to the tank. Hence if g_n is the amount of oil in the tank at the end of year n, then $g_n = ag_{n-1} + b$, where $0 < a < 1, b > 0, n \geq 1$.

a) Find g_1, then g_2, then g_3, then g_4, then observe the pattern and use the formula for the sum of a finite geometric series to find an expression for g_n in terms of g_0, a, b.

b) Find the amount of oil in the tank in the limit $n \to \infty$. Does the answer depend on g_0?