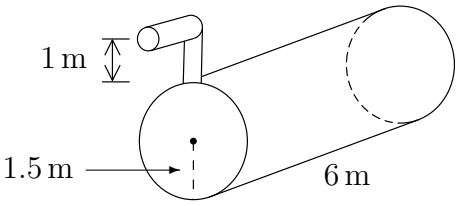
Math 156 Applied Honors Calculus II Fall 2024

hw4 , due: Tuesday, September 24 at 4pm



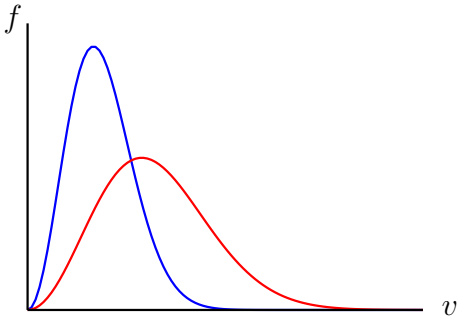
1. A cylindrical tank full of water is lying on its side. Find the work done in pumping the water to the top of the outlet. The water density is $\rho = 1000\,\text{kg/m}^3$. Show the steps when you evaluate the integral. Express the answer in MJ.

2. An object with mass m is moving in a straight line subject to a force $f(x)$, where x is the object's position. Let $v(x)$ be the object's velocity as a function of position.

a) Show that the work done in moving the object from x_0 to x_1 is equal to the change in the object's kinetic energy, $W = \int_{x_0}^{x_1} f(x)dx = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2$, where $v_0 = v(x_0), v_1 = v(x_1)$. (hint: $f(x) = ma = m\frac{dv}{dt}$, where a is acceleration and t is time, then $\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v$)

b) How many foot-pounds of work does it take to pitch a baseball at 90 mi/hr? Assume the baseball weighs 5 oz. (note: 1 lb = 16 oz, 1 mile = 5280 ft, $g = 32\,\text{ft/sec}^2$, 1 hr = 3600 sec; also recall that weight = force = mass × acceleration)

3. Sketch the function; determine whether the integral converges or diverges; if it converges, find the value. a) $\int_{-\infty}^{\infty} xe^{-x^2}dx$ b) $\int_{-\infty}^{\infty} e^{-|x|}dx$ c) $\int_0^1 \frac{dx}{x\sqrt{x}}$



4. According to Maxwell-Boltzmann kinetic theory, the average speed of gas molecules in a container is $\bar{v} = \int_0^{\infty} vf(v)dv$, where $f(v) = \frac{4}{\sqrt{\pi}}\left(\frac{M}{2RT}\right)^{3/2} v^2 e^{-Mv^2/(2RT)}$; M is the molecular weight of the gas, R is the gas constant, and T is the gas temperature.

a) The figure shows $f(v)$ for two values of temperature, $T_1 < T_2$; copy the figure and label each curve with the corresponding T. Give a mathematical reason to justify your answer.

b) Show that $\bar{v} = \left(\frac{8RT}{\pi M}\right)^{1/2}$. (hint: substitute $x = Mv^2/2RT$)

c) The boiling point of diatomic oxygen (O_2) is 90 K (degrees kelvin). A container is filled with O_2 where the average speed of the molecules is 300 m/s. Is the oxygen in the liquid state or gas state? Take $M = 32$ g/mol, $R = 8.314$ J/(K-mol), where mol = 1 mole.

5. The Gamma function $\Gamma(x)$ is defined by $\Gamma(x) = \int_0^{\infty} t^{x-1}e^{-t}dt$.

a) Find $\Gamma(1), \Gamma(2), \Gamma(3)$. b) Show that $\Gamma(n+1) = n\Gamma(n)$ for $n \geq 1$; use this to compute $\Gamma(4)$.

note : It follows from (b) that $\Gamma(n+1) = n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1 = n!$ ("n factorial").

6. a) Show that $\dfrac{1}{\sqrt{x^2+1}} \sim \dfrac{1}{x}$ as $x \to \infty$; use the definition of $f(x) \sim g(x)$ as $x \to \infty$ on page 19 in the notes.

b) Sketch the graphs of $\dfrac{1}{\sqrt{x^2+1}}$ and $\dfrac{1}{x}$ for $x \geq 0$ on the same plot and label each curve. Your sketch should reflect what was shown in part (a). Do the curves intersect? Justify your answer.

announcement The 1st midterm exam is on Wednesday, September 25 at 6-7:30pm in a room tba. If you need accommodation, please arrange that with your instructor. The exam covers 1.1 sigma notation, 1.2 area, 1.3 definite integral, 1.4 FTC, 1.5 work, 1.6 improper integrals, plus the homework and lecture notes. We will supply the exam booklets. At the end of the exam, scan your solutions and upload them into Gradescope. A review sheet with sample problems will be distributed soon. Calculators are not allowed, but you may use one sheet of paper (one side) for handwritten notes.