1. The Laplace transform of a function $f(t)$ is a new function $F(s) = \int_0^\infty f(t)e^{-st}dt$; this construction is used in solving differential equations (Math 216/286/316). Find the Laplace transform $F(s)$ of the following functions.  
   a) $f(t) = 1$  
   b) $f(t) = e^t$  
   c) $f(t) = t$  

   note: To ensure the integral converges we assume $s > 0$ in (a,c) and $s > 1$ in (b).

2. Consider the integral $\int_0^\infty \left( \frac{x}{x^2+1} - \frac{c}{3x+1} \right)dx$, where $c > 0$ is a constant.

   a) Show that writing it as $\int_0^\infty \frac{x}{x^2+1}dx - \int_0^\infty \frac{c}{3x+1}dx$ yields $\infty - \infty$, which is undefined.

   b) Consider the functions $\frac{x}{x^2+1}$ and $\frac{c}{3x+1}$; note that the 1st function is asymptotic to $\frac{1}{x}$ as $x \to \infty$; for what value of $c$ is the 2nd function also asymptotic to $\frac{1}{x}$ as $x \to \infty$?

   c) Let $c$ have the value found in (b); evaluate the original integral by combining the two antiderivatives; in this way we make sense of $\infty - \infty$.

   d) The value of the integral obtained in (c) is negative. Give a reason to justify this.

3. Three students order a 14 inch pizza, but instead of slicing it the usual way, they slice it by two parallel cuts, at $x = a$ and $x = -a$. Find a formula for $a$ ensuring that each student gets the same amount of pizza. Evaluate the integrals in the formula by the FTC, solve for $a$ using a calculator, and express the answer in inches.

4. a) Using a calculator compute $\left( 1 + \frac{1}{n} \right)^n$ for $n = 1, 10, 10^2, 10^3, 10^4$; keep 5 decimal digits.

   b) Evaluate $L = \lim_{n \to \infty} \left( 1 + \frac{1}{n} \right)^n$. (hint: consider $\ln L$)

5. The van der Waals equation of state of a gas gives the pressure $P$ in terms of the volume $V$ and temperature $T$ as $P = \frac{RT}{V-b} - \frac{a}{V^2}$, where $R$ is the ideal gas constant, and $a, b$ are positive constants depending on the type of molecules in the gas. In an isothermal change of state, the temperature $T$ is constant, and the work done in compressing the gas from volume $V_1$ to volume $V_2$ is $W = \int_{V_1}^{V_2} PdV$. Evaluate the integral and find $W$ in terms of $V_1, V_2, T, R, a, b$.

6. Consider a circular sector with radius $r$ and angle $\theta$ in the $xy$-plane. In class we used a scaling approach to show that the curved sector edge has length $L = r\theta$ and the area of the sector is $A = \frac{1}{2}r^2\theta$. Rederive these results using the formulas for the arclength of a graph, $L = \int_a^b \sqrt{1 + (f'(x))^2} \, dx$, and the area under a graph, $A = \int_a^b f(x) \, dx$. In each case you need to find the appropriate $f(x), a, b$, and evaluate the formulas to obtain $L, A$ in terms of $r, \theta$. In the case of the area, write $A = A_1 + A_2$, where $A_1$ is the area of a triangle and $A_2$ is the area under the graph of a function. (hint: in this problem, $\theta$ is a given parameter; when you apply trig substitution you must use a different symbol for the angle, e.g. $\phi$)