Math 156 Applied Honors Calculus II Fall 2024

hw5 , due: Friday, October 4 at 4pm

1. The Laplace transform of a function $f(t)$ is a new function $F(s) = \int_0^\infty f(t)e^{-st}dt$, which is useful in solving differential equations (Math 216/286/316). Find the Laplace transform $F(s)$ of the following functions. a) $f(t) = 1$ b) $f(t) = t$ c) $f(t) = e^t$

note : To ensure the integral converges, we assume $s > 0$ in (a,b) and $s > 1$ in (c).

2. Consider the integral $\int_0^\infty \left(\frac{x}{x^2+1} - \frac{c}{3x+1}\right)dx$, where $c > 0$ is a constant.

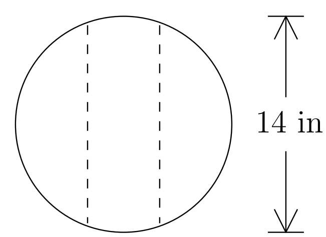
a) Show that writing the integral as $\int_0^\infty \frac{x}{x^2+1}dx - \int_0^\infty \frac{c}{3x+1}dx$ yields $\infty - \infty$, which is undefined.

b) Consider the functions $\frac{x}{x^2+1}$ and $\frac{c}{3x+1}$; note that the 1st function is asymptotic to $\frac{1}{x}$ as $x \to \infty$; for what value of c is the 2nd function also asymptotic to $\frac{1}{x}$ as $x \to \infty$?

c) Let c have the value found in (b); evaluate the original integral by combining the two antiderivatives; in this way we make sense of $\infty - \infty$.

d) The value of the integral obtained in (c) is negative. Give a reason to justify this. (hint: sketch the graphs of the functions)

3. Three students order a 14 inch pizza, but instead of slicing it the usual way, they slice it by two parallel cuts, at $x = a$ and $x = -a$. Find a formula for a ensuring that each student gets the same amount of pizza. Evaluate the integrals in the formula by the FTC, show the steps in deriving the antiderivative, solve for a using a calculator, and express the answer in inches.



4. a) Using a calculator compute $\left(1 + \frac{1}{n}\right)^n$ for $n = 1, 10, 10^2, 10^3, 10^4$; keep 5 decimal digits.

b) Evaluate $L = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$. (hint: consider $\ln L$)

5. The van der Waals equation of state of a gas gives the pressure P in terms of the volume V and temperature T as $P = \frac{RT}{V-b} - \frac{a}{V^2}$, where R is the ideal gas constant, and a, b are positive constants depending on the type of molecules in the gas. In an isothermal change of state the temperature T is constant and the work done in compressing the gas from volume V_1 to volume V_2 is $W = \int_{V_1}^{V_2} P dV$. Evaluate the integral and find W in terms of V_1, V_2, T, R, a, b.

6. Consider a circular sector with radius r and angle θ in the xy-plane. In class we used a scaling argument to show that the length of the curved sector edge is $L = r\theta$ and the area of the sector is $A = \frac{1}{2}r^2\theta$. Rederive these results using the formulas for the arclength of a graph, $L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$, and the area under a graph, $A = \int_a^b f(x)dx$. In each case you need to find the appropriate $f(x), a, b$, and evaluate the formulas to obtain L, A in terms of r, θ. In the case of the area, write $A = A_1 + A_2$, where A_1 is the area of a triangle and A_2 is the area under the graph of a function. (hint: in this problem θ is a given parameter; when you apply trig substitution you must use a different symbol for the angle, e.g. ϕ)