Math 156 Applied Honors Calculus II Fall 2023

hw8 , due: Tuesday, October 31 at 4pm

1. The waiting time in a fast-food restaurant is exponentially distributed with mean 2.5 minutes. a) Find the probability that a customer is served in the first 2 minutes. b) Find the probability that a customer waits 4 minutes or more. c) The manager wants to advertise that anyone waiting more than M minutes will receive a free meal. What should the value of M be to avoid giving free meals to more than 2% of the customers?

2. When the chemical reaction $N_2O_5 \to 2NO_2 + \frac{1}{2}O_2$ takes place at 45° C, the concentration of N_2O_5 (dinitrogen pentoxide) decays in time as described by the differential equation $\frac{d}{dt}[N_2O_5] = -0.0005 \cdot [N_2O_5]$, where the concentration $[N_2O_5]$ is measured in units of mole/liter and time t is measured in seconds. a) Find an expression for $[N_2O_5]$ after t seconds assuming the initial concentration is c_0. b) How long does it take for the N_2O_5 concentration to fall to 90% of c_0?

3. a) How long does it take for an investment to double in value if the annual interest rate is 6% and the interest is compounded continuously? b) Find the equivalent annual interest rate assuming the interest is compounded annually.

4. A tank initially contains 1000 L of brine with 15 kg of dissolved salt, and pure water starts pouring into the tank at a rate of 10 L/min. The solution is well mixed and it drains from the tank at the same rate that the pure water enters. Find the amount of salt in the tank after (a) t minutes, (b) 20 minutes.

5. Glucose is introduced into a patient's bloodstream at a rate r [mg/s], and once there it is converted to other substances and is depleted at a rate proportional to the amount present with depletion constant k [1/s]. Assume $r > 0, k > 0$. Let $g(t)$ [mg] be the amount of glucose in the bloodstream at time t. a) Find the differential equation for $g(t)$. b) Let $g_0 \geq 0$ be the initial amount of glucose in the bloodstream; find $g(t)$ in terms of g_0, r, k. c) Find $G = \lim_{t\to\infty} g(t)$. d) Sketch $g(t)$ for $t \geq 0$; consider two cases, $g_0 < G, g_0 > G$; indicate G on the sketch.

6. a) Show that $\sinh^{-1}x = \ln\left(x + \sqrt{x^2+1}\right)$. (hint: set $x = \sinh y$ and solve for y)

b) In class we showed that $\displaystyle\int \frac{dx}{\sqrt{x^2+1}} = \ln(x + \sqrt{x^2+1})$ using the trig substitution $x = \tan\theta$ (page 20); now rederive this using the hyperbolic trig substitution $x = \sinh y$.

7. (a) Show that $f(x) = f(a) + f'(a)(x-a) + \int_a^x (x-t)f''(t)\,dt$. (hint: start from the integral and apply integration by parts with $u = x - t$, $dv = f''(t)\,dt$.)

b) Given a function $f(x)$ and a point a, define $T_1(x) = f(a) + f'(a)(x-a)$; $T_1(x)$ is a linear function of x called the <u>Taylor polynomial of degree 1</u>. Show that $T_1(a) = f(a)$, compute $T_1'(x)$ and show that $T_1'(a) = f'(a)$; hence $T_1(x)$ is tangent to $f(x)$ at $x = a$, and we say that $T_1(x)$ is a <u>linear approximation</u> to $f(x)$ at $x = a$.

c) Using (a,b) derive the <u>error bound</u>, $|f(x) - T_1(x)| \leq \frac{1}{2}M_2|x-a|^2$, where $M_2 = \max|f''(t)|$. (hint: (a,b) $\Rightarrow f(x) = T_1(x) + \int_a^x (x-t)f''(t)\,dt$, then follow the steps from hw7, problem 8)

d) Let $f(x) = e^x, a = 0$. Find $T_1(x)$. Sketch $f(x), T_1(x)$ on the same graph around $x = a$.

e) Make a table with the following format. column 1: $|x-a|$, column 2: $|f(x) - T_1(x)|$. Take $f(x) = e^x, a = 0$ and fill it in for $x = 1, 1/2, 1/4, 1/8$ using a calculator. When $|x-a|$ is reduced by a factor of $\frac{1}{2}$, by approximately what factor is the error $|f(x) - T_1(x)|$ reduced? Is this consistent with the error bound derived in part (c)?

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