Math 156 Applied Honors Calculus II Fall 2024

hw11 , due: Friday, December 6 at 4pm

1. Consider the power series $\sum_{n=0}^{\infty} \frac{(x-1)^n}{2^n}$. Find the interval of convergence; find the function $f(x)$ represented by the series; sketch the graph of $f(x)$ for $-3 \le x \le 7$; indicate the center of the series and interval of convergence on the x-axis; indicate any asymptotes with a dashed line.

2. Find $T_1(x)$ and $T_3(x)$, the 1st and 3rd degree Taylor polynomials for $f(x) = \sin x$ at $a = \pi$. Sketch the graphs of $f(x), T_1(x), T_3(x)$ on the same plot for $-2\pi \le x \le 2\pi$ and label each one.

3. Express $\int_0^{1/2} \frac{dx}{1+x^6}$ as a series; give the first 3 terms and the general term. How many terms are needed in the partial sum to ensure the error is less than 10^{-5}? Use an error bound derived in class.

4. We can approximate $\sqrt{10}$ using the fact that it is close to $\sqrt{9} = 3$.

(a) Find $T_1(x)$, the 1st degree Taylor approximation for $f(x) = \sqrt{x}$ at $a = 9$, and evaluate $T_1(10)$.

(b) Repeat for $T_2(x)$, the 2nd degree Taylor approximation.

Express the answers as a rational number (i.e. m/n, where m, n are integers) and in decimal form with eight decimal digits. Then find the exact value of $\sqrt{10}$ using a calculator and compute the error in each case; does the Taylor approximation become more accurate as the degree increases?

5. The Arrhenius equation, $k = Ae^{-E/RT}$, gives the rate constant k of a chemical reaction in terms of the temperature T, where A is a constant, E is the activation energy, and R is the universal gas constant (these are all positive). Find $\lim_{T\to 0} k$, $\lim_{T\to\infty} k$. Sketch the graph of k versus T for $T \ge 0$ (hint: see page 66 in the notes). As the temperature increases, does the reaction occur slower or faster?

6. A non-reflecting object that absorbs all incident radiation is called a blackbody and in thermal equilibrium it emits radiation with energy density $f(\lambda)$, where λ is the radiation wavelength. The Rayleigh-Jeans law for energy density is $f_{RJ}(\lambda) = 8\pi k_{\mathrm{B}} T \lambda^{-4}$, where k_{B} is Boltzmann's constant and T is the object's temperature; it agrees with experiments for long wavelength, $\lim_{\lambda\to\infty} f_{RJ}(\lambda) = 0$, but not for short wavelength; experiments show that $\lim_{\lambda\to 0} f(\lambda) = 0$, but $\lim_{\lambda\to 0} f_{RJ}(\lambda) = \infty$; this is called the ultraviolet catastrophe. In 1900 Max Planck used the novel idea of energy quantization to derive a different expression, $f_P(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda k_{\mathrm{B}} T} - 1}$, where h is Planck's constant and c is the speed of light.

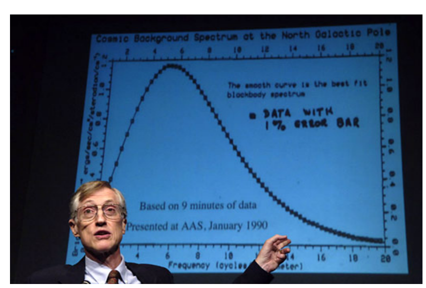
a) Show that $\lim_{\lambda\to\infty} f_P(\lambda) = 0$ and $\lim_{\lambda\to 0} f_P(\lambda) = 0$; hence Planck's law agrees with experiments in the long and short wavelength limits (hint: set $x = hc/\lambda k_{\mathrm{B}} T$).

b) Show that $f_P'(0) = 0$ (hint: use the definition, see page 66 in the notes)

c) Show that $f_P(\lambda) \sim f_{RJ}(\lambda)$ for $\lambda \to \infty$; recall that this means $\lim_{\lambda\to\infty} f_P(\lambda)/f_{RJ}(\lambda) = 1$.

d) Show that $f_P(\lambda) < f_{RJ}(\lambda)$ for $\lambda \ge 0$.

e) Sketch $f_P(\lambda), f_{RJ}(\lambda)$ for $\lambda \ge 0$ on the same plot and label them.



Planck's discovery is described in the article, "Max Planck: the reluctant revolutionary"; see link on Canvas site. In 2006 John Mather and George Smoot won the Nobel Prize in Physics, "for their discovery of the blackbody form … of the cosmic microwave background radiation"; the photo shows Mather with a plot of the experimental data in excellent agreement with Planck's law.

announcement The final exam is on Thursday, December 12 at 8-10am (sections 1, 2, 5 in 1200 CHEM, sections 3, 4 in 1300 CHEM). It covers the entire course; calculators are not allowed; we will supply the exam booklets; the exam will be uploaded to Gradescope; you may use 2 sheets of notes (e.g. 2 sides of one page).