Math 371   Winter 2013   Homework 1   due: Tuesday January 22

Please write neatly, explain your answers, and staple the sheets together.

0. (optional) Give a brief description of your academic background and interests. If you work in a lab or research group, please give your supervisor's name and describe your project. One paragraph is fine.

1. a) Convert $(2013)_{10}$ to base 2.   b) Convert $(110110.001)_2$ to base 10.

2. The floating point representation of a real number has the form $x = \pm(0.d_1d_2 \ldots d_n)_\beta \cdot \beta^e$, where $d_1 \neq 0, 0 \leq d_i \leq \beta - 1, -M \leq e \leq M$.

a) Suppose $\beta = 2, n = 4, M = 3$. How many different numbers can be represented in this system?

b) Find $\text{fl}(\sqrt{2})$, the floating point representation of $\sqrt{2}$ in this system. Convert the result to decimal form. How large is the roundoff error?

c) In class we saw that $x_{\max} = 7.5, x_{\min} = 0.0625$ (these are the largest and smallest positive numbers that can be represented in this system). Suppose we switch to a system with $n = 5$. Find the new values of $x_{\max}, x_{\min}$.

3. Matlab gives $\texttt{pi} = 3.141592653589793$ and $355/113 = 3.141592920353983$. All the digits shown are correct. (Use the command $\texttt{format long}$ to see all the digits). Matlab also gives $\texttt{pi} - (355/113) = -2.667641894049666\text{e}{-07}$; do you trust all the digits in this result? Explain your answer.

4. Let $f(x) = 1 - \sin x$ and $g(x) = \dfrac{\cos^2 x}{1 + \sin x}$.

a) Show that $f(x) = g(x)$, assuming that $1 + \sin x \neq 0$.

b) Which function should be used when x is near $\pi/2$? Explain.

c) Which function should be used when x is near $3\pi/2$? Explain.

5. Consider the equation $x^2 + 25x + 0.1 = 0$.
a) Solve for the roots using the quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. First use Matlab or a calculator and then use 4-digit arithmetic as in class. To implement 4-digit arithmetic, use Matlab or a calculator, but round each intermediate step to 4 digits. Explain the results.

b) Repeat using the other form of the quadratic formula, $x = \dfrac{2c}{-b \pm \sqrt{b^2 - 4ac}}$. Do the results change? Explain.

note: Here are some Matlab resources.

1. a short tutorial with links to longer ones
       www.math.lsa.umich.edu/~krasny/matlab.pdf

2. MathWorks webpage with videos and tutorials.
       www.mathworks.com/academia/student_center/tutorials/launchpad.html

3. "The Origins of MATLAB", by Cleve Moler (creator of MATLAB)
       www.mathworks.com/company/newsletters/news_notes/clevescorner/dec04.html