Math 371 Winter 2013 Homework 2 due: Tuesday January 29

Some problems have a yes/no answer, but to obtain full credit you need to explain your answer.

1. Let $f(x) = \sqrt{1+x^2} - 1$.

a) Evaluate $f(x)$ for $x = 0.1$ using 4-digit arithmetic. Show all intermediate steps.

b) Show that $f(x) = x^2/(\sqrt{1+x^2}+1)$ and repeat part (a).

c) Matlab gives $f(0.1) = 0.004987562112089$. Which result agrees best with Matlab, (a) or (b)?

2. Let $f(x) = \sin x$ and take $x = \frac{\pi}{4}$, so that $f'(x) = f'(\frac{\pi}{4}) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} = 0.70710678$.

a) The forward difference approximation for $f'(x)$ is $D_+f(x) = \dfrac{f(x+h)-f(x)}{h}$.

Present a table in the format below; take $h = 0.1, 0.05, 0.025, 0.0125$; the first line for $h = 0.1$ is given and you must fill in the entries for the remaining values of h. Indicate the limit of each column, as we did in class.

h	$D_+f(x)$	$\lvert f'(x) - D_+f(x)\rvert$	$\dfrac{\lvert f'(x) - D_+f(x)\rvert}{h}$	$\dfrac{\lvert f'(x) - D_+f(x)\rvert}{h^2}$	$\dfrac{\lvert f'(x) - D_+f(x)\rvert}{h^3}$
0.1	0.67060297	0.03650381	0.3650381	3.650381	36.50381

In class we showed using Taylor series that $D_+f(x) = f'(x) + \frac{1}{2}f''(x)h + \cdots$, which implies that $D_+f(x)$ is 1st order accurate (i.e. the truncation error is $O(h)$). Is the table consistent with this result?

b) The centered difference approximation for $f'(x)$ is $D_0f(x) = \dfrac{f(x+h)-f(x-h)}{2h}$.

Present a table for $D_0f(x)$ in the above format. From the results in these tables, which approximation is more accurate, $D_+f(x)$ or $D_0f(x)$?

c) Using Taylor series, show that $D_0f(x) = f'(x) + ch^2 + \cdots$ for some contant c, which is independent of h. To derive this result, you need to retain sufficiently many terms in the Taylor series. What is the order of accuracy of $D_0f(x)$? Is this evident from the table?

d) Modify the Matlab code given in class to plot the error in $D_+f(x)$ and $D_0f(x)$ for step size $h = 1/2^{(j-1)}$ with $j = 1:65$. Use log scales for the error $|f'(x) - Df(x)|$ and the step size h. Plot both cases on the same graph (to do this in Matlab, type `hold on` after the first `loglog` command). Explain the results.

3. Consider $f(x) = x^2 - 5$. Since $f(2) = -1 < 0, f(3) = 4 > 0$, it follows that $f(x)$ has a root r in the interval $[2,3]$. Compute an approximation to r by the following methods.

a) bisection method, starting interval $[a,b] = [2,3]$

b) fixed-point iteration with $g_1(x) = 5/x, g_2(x) = x - (x^2-5), g_3(x) = x - \frac{1}{3}(x^2-5)$, $x_0 = 2.5$

c) Newton's method, $x_0 = 2.5$

Take 10 steps in each case. Present a table with the following format; column 1: n, column 2: x_n, column 3: $|r - x_n|$. Do the results agree with the theory discussed in class?

announcement

The first quiz is on Thursday, February 7. It will start promptly at 12:10pm and will be 20 minutes long. The quiz is closed book and will cover material up to and including the previous class. Please bring a calculator to do arithmetic. The best way to prepare is to review the lecture notes and homework.