Math 371 Winter 2013 Homework 9 due: Tuesday April 23

There are several Matlab exercises on this assignment; it is not necessary to turn in the code.

1. Consider $f(x) = \sin x$ for $-4\pi \le x \le 4\pi$. Find the Taylor series for $f(x)$ about $x = 0$ up to the x^7-term. Using Matlab, plot $f(x)$ and the Taylor polynomials $p_n(x)$ of degree $n = 1, 3, 5, 7$. Use `subplot` and put each Taylor polynomial in a different frame. Use the command `axis([-4*pi 4*pi -2 2])` to set the limits on the axes. Label each curve. Turn in the resulting plot.

The Taylor polynomial $p_n(x)$ is an approximation to $f(x)$. Answer the following questions and justify your answer.

a) For a given value of n, is the approximation valid for all values of x?

b) Does the approximation improve as the degree n increases?

2. Let $f(x) = e^{-|x|}$ and consider the three points $x_0 = -1, x_1 = 0, x_2 = 1$.

a) Find Newton's form for the interpolating polynomial $p_2(x)$.

b) Find the standard form for the interpolating polynomial $p_2(x)$.

c) Plot $f(x)$ and $p_2(x)$ on the same graph and label each curve. Sketch by hand or use Matlab.

d) Compute $\int_{-1}^{1} f(x)dx$ and $\int_{-1}^{1} p_2(x)dx$.

3. Assume $f(x)$ is given and let $x_0, x_1, \ldots, x_n$ be $n+1$ distinct points. In class we discussed the interpolating polynomial $p_n(x)$ of degree $\le n$ that interpolates $f(x)$ at the given points, i.e. such that $p_n(x_i) = f(x_i)$ for $i = 0 : n$. The template on the back of this sheet plots $f(x)$ and $p_n(x)$ for $n = 4, 8, 16$, for uniform points and Chebyshev points. Your assignment is to fill in the template. First consider the case $f(x) = \frac{1}{1+25x^2}$ and make sure your results agree with those shown in the lecture notes (chapter 5, page 6). Then run the code for the functions $f(x) = e^{-|x|}$ and $f(x) = e^{-x^2}$, and turn in the outplot plots. How do these results compare with the results for $f(x) = \frac{1}{1+25x^2}$? Can you explain any differences?

announcements

1. On Tuesday April 16, both sections will meet in 133 Chrysler.

2. The online teaching evaluations will be available from Thursday April 11 to Wednesday April 24. Please complete the evaluations; they provide valuable feedback from students to instructors.

3. The final exam is on Wednesday, May 1, 1:30-3:30pm. Section 1 (Krasny) will meet in 1017 DOW and Section 2 (Wang) will meet in 1005 DOW. The exam will cover the entire course, though material since the midterm exam will be emphasized. A review sheet will be distributed soon. You may use two pages of handwritten notes (i.e. two sides of one sheet, 8.5 in $\times$ 11 in) and a calculator to do arithmetic. Explain your reasoning and justify your answers to receive full credit. We will supply the exam booklets.

```
function m371_interpolation
%
% This file plots the interpolating polynomial p_n(x) for f(x) at n+1 points
% in the interval [-1,1] using the Matlab commands polyfit and polyval.
%
% itype = 1 : uniform points, itype = 2 : Chebyshev points
%
clear; clf;
%
for k = 1:6
    if k==1; n = 4; itype=1; text='uniform points , n=4'; end
    if k==2; n = 4; itype=2; text='Chebyshev points , n=4'; end
    if k==3; n = 8; itype=1; text='uniform points , n=8'; end
    if k==4, n = 8; itype=2; text='Chebyshev points , n=8'; end
    if k==5, n = 16; itype=1; text='uniform points , n=16'; end
    if k==6, n = 16; itype=2; text='Chebyshev points , n=16'; end
%
    if itype==1; h = ...; x = ...; end
    if itype==2; h = ...; x = ...; end
%
    f = ...;
    coeff = polyfit(x,f,n); % polyfit computes the coefficients of p_n(x)
%
% Plot f(x) and p_n(x) on a fine mesh.
%
    h = 0.001; x = -1.5:h:1.5;
    f = ...;
    p = polyval(coeff,x); % polyval evaluates p_n(x)
%
    subplot(3,2,k)
    plot(x,f,x,p,'--'); axis([-1.5 1.5 -1.5 1.5])
    title(text)
end
```