Math 471 Fall 2009 Homework 2 due: Mon Oct 5

When numerical answers are required, you may use Matlab or a calculator, unless other instructions are given.

1. The forward and backward finite-difference operators are defined by

$$D_+f(x) = \frac{f(x+h) - f(x)}{h}, \quad D_-f(x) = \frac{f(x) - f(x-h)}{h}.$$

a) Show that $D_+D_-f(x) = \dfrac{f(x+h) - 2f(x) + f(x-h)}{h^2}$.

b) Show that $D_+D_-f(x) = f''(x) + O(h^2)$ and find the asymptotic error constant.

chapter 2, rootfinding

2. Consider $f(x) = x^2 - 5$. Since $f(2) < 0, f(3) > 0$, it follows that $f(x)$ has a root p in the interval $[2, 3]$. Compute an approximation to p by the following methods. Take 10 steps in each case. Use Matlab and print the answers to 15 digits.

a) bisection method, starting interval $[a, b] = [2, 3]$

b) fixed-point iteration with $g_1(x) = 5/x$ and $g_2(x) = x - f(x)/3$, starting value $x_0 = 2.5$

c) Newton's method, starting value $x_0 = 2.5$

Present the results in a table with columns as below for each method. Do the results agree with the theory discussed in class?

column 1 : n (step)
column 2 : x_n (approximation)
column 3 : $f(x_n)$ (residual)
column 4 : $|p - x_n|$ (error)

3. In class we discussed the example, "Volume of Chlorine Gas" on page 102. This example uses Newton's method to compute the volume of a gas given by van der Waal's equation of state, where the initial guess V_0 is given by the ideal gas law. We saw that V_0 has 2 correct digits and V_1 has 5 correct digits. How many correct digits does V_2 have?

4. Consider the following system of nonlinear equations.

$$f(x, y) = (x-1)^2 + y^2 - 4 = 0, \quad g(x, y) = xy - 1 = 0$$

This corresponds to finding the intersection of a circle and a hyperbola. Find an approximate solution using Newton's method for systems. Take two steps starting from $(x_0, y_0) = (3, 0)$. Present the iterates (x_i, y_i) and residual values $f(x_i, y_i), g(x_i, y_i)$ for $i = 0, 1, 2$.

chapter 3, linear algebra

5. page 148, problem 4a,b , 7a (warmup exercise on matrices)

6. page 149, problem 14b (hint: it is sufficient to show that $AA^{-1} = I$)

7. page 157, problem 1 (Gaussian elimination)