

Math 471 Fall 2009 Homework 3 due: Wed Oct 21

chapter 3, linear algebra

1. Which of the following matrices are invertible? Justify your answer.

(a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \\ 0 & 3 & 3 \end{pmatrix}$

2. page 159, problem 13 (electric circuit, solve by Gaussian elimination)

3. page 169, problem 14, solve the system three ways:

- (a) Gaussian elimination with no pivoting, 3 decimal digit arithmetic with rounding
- (b) Gaussian elimination with partial pivoting, 3 decimal digit arithmetic with rounding
- (c) Matlab backslash command

4. page 180, problems 1, 2b (vector norms)

5. Let $A = \begin{pmatrix} -2 & 1 \\ 2 & 0 \end{pmatrix}$.

(a) Find $\frac{\|Ax\|_\infty}{\|x\|_\infty}$ for the following vectors. $x_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $x_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $x_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(b) Find a vector x such that $\frac{\|Ax\|_\infty}{\|x\|_\infty} = \|A\|_\infty$.

6. Let $A = \begin{pmatrix} 1.2969 & 0.8648 \\ 0.2161 & 0.1441 \end{pmatrix}$, $b = \begin{pmatrix} 0.8642 \\ 0.1440 \end{pmatrix}$, $x = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$, $x_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $x_2 = \begin{pmatrix} 0.9911 \\ -0.4870 \end{pmatrix}$.

a) Show that x is the exact solution of $Ax = b$.

b) Think of x_1, x_2 as approximations to the exact solution x . Compute the errors e_1, e_2 and residuals r_1, r_2 , corresponding to x_1, x_2 .

c) Find $\|A\|_\infty, \|A^{-1}\|_\infty, \kappa_\infty(A)$.

d) In class we proved the following theorem relating the relative error, relative residual, and condition number.

$$\frac{\|e\|}{\|x\|} \leq \kappa(A) \frac{\|r\|}{\|b\|}$$

Show that this result holds for the approximate solutions x_1, x_2 given above.

7. Derive the following result, which was stated in class.

$$\left. \begin{array}{l} Ax = b \\ \tilde{A}\tilde{x} = b \end{array} \right\} \Rightarrow \frac{\|x - \tilde{x}\|}{\|\tilde{x}\|} \leq \kappa(A) \frac{\|A - \tilde{A}\|}{\|A\|}$$

note : This result says that in solving a linear system $Ax = b$, the condition number of the matrix controls the relative error in the solution due to perturbations in the matrix.

announcement

The midterm exam is on Friday October 30 in class. A review sheet with sample problems will be distributed before the exam. You may use a non-programmable calculator to do arithmetic, but to receive full credit you must show all intermediate steps. You may use one sheet of notes (i.e. one side of one page, 8.5 in \times 11 in). Exam booklets will be provided.