Math 471 Fall 2009 Homework 4 due: Fri Oct 30

section 3.5, LU factorization

1. Let A be a 3×3 matrix. Suppose we apply LU factorization with partial pivoting and obtain $E_2 P_2 E_1 P_1 A = U$, where U is upper triangular and

$$E_1 = \begin{pmatrix} 1 & 0 & 0 \\ -m_{21} & 1 & 0 \\ -m_{31} & 0 & 1 \end{pmatrix}, \; E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -m_{32} & 1 \end{pmatrix}, \; P_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \; P_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

(a) Compute $\tilde{E}_1 = P_2 E_1 P_2$.
(b) Show that $P_2 E_1 = \tilde{E}_1 P_2$. Note that this implies $E_2 \tilde{E}_1 P_2 P_1 A = U$.
(c) Compute $P = P_2 P_1$ and $L = \tilde{E}_1^{-1} E_2^{-1}$.
(d) Show that $PA = LU$.
(e) Find P, L, U such that $PA = LU$ and use the factorization to solve $Ax = b$, where
$$A = \begin{pmatrix} 0 & 2 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{pmatrix}, \; b = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}.$$

section 3.8, iterative methods

2. Consider the linear system, $2x_1 + x_2 = 1, x_1 + 2x_2 = -1$.
(a) Write the system in matrix form and solve it by LU factorization.
(b) Write out Jacobi's method in component form and take three steps starting from initial guess $x_0 = (0,0)^T$. Present the results in a table with the following format.

column 1: k (iteration step)
column 2: $x_1^{(k)}$ (1st component of computed solution vector at step k)
column 3: $x_2^{(k)}$ (2nd component of computed solution vector at step k)
column 4: $||e_k||_\infty$ (error norm at step k)
column 5: $||e_k||_\infty / ||e_{k-1}||_\infty$ (ratio of successive error norms at step k)

Find the iteration matrix B_J and compute $||B_J||_\infty, \rho(B_J)$. Does the method converge?
(c) Repeat part (b) for Gauss-Seidel.
(d) Repeat part (b) for optimal SOR.

Computing Project 1 , due: Mon Nov 9

section 8.1, two-point boundary value problem

These exercises should be done in Matlab, using the finite-difference scheme and tridiagonal LU solver discussed in class. The code should not create the full matrix, but should instead use linear arrays containing the nonzero matrix elements, as discussed in class.

1. Consider the 2-point BVP, $-\epsilon y'' + y = 2x + 1, 0 \le x \le 1, y(0) = 0, y(1) = 0, \epsilon = 10^{-3}$. Show that the exact solution is $y(x) = 2x + 1 - (\sinh \frac{1-x}{\sqrt{\epsilon}} + 3 \sinh \frac{x}{\sqrt{\epsilon}}) / \sinh \frac{1}{\sqrt{\epsilon}}$. Plot the exact solution and the numerical solution for $h = \frac{1}{32}$ (the results for larger h were shown in class).

2. page 672, problem 15a (deflection of a beam)
The deflection of a beam's centerline, $u(x)$, satisfies the equation, $u'' - \frac{T}{EI} u = -\frac{w}{2EI} x(L-x)$, subject to boundary conditions $u(0) = u(L) = 0$. The textbook asks you to compute the beam deflection at 1-inch intervals, and you should do two additional cases, 2-inch and 4-inch intervals (so we can assess the accuracy of the results). Plot all three cases in a single plot (use different symbols to distinguish the cases). Give the maximum beam deflection computed in each case.