

chapter 5, polynomial approximation and interpolation

1. a) Find the Taylor series for $f(x) = \sin x$ about $x = 0$.
b) Using Matlab, plot the Taylor polynomials $p_n(x)$ of degree $n = 1, 3, 5, 7$ on the same plot on the interval $-4\pi \leq x \leq 4\pi$. Also plot the original function $f(x) = \sin x$. Label each curve. We view the Taylor polynomial as an approximation to the original function. Describe in words what happens to the approximation as the degree of the polynomial increases.
2. In class we saw that the Taylor series for $f(x) = \frac{1}{1+25x^2}$ about $x = 0$ converges for $|x| < \frac{1}{5}$. Now we consider the expansion about $x = \frac{1}{5}$.
a) Let $T_i(x)$ be the Taylor polynomial for $f(x)$ about $x = \frac{1}{5}$. Using Matlab, plot $f(x)$ and $T_i(x)$ for $i = 0, 1, 2, 3$ on the interval $-1 \leq x \leq 1$. How do the results compare with the results given in class?
b) On what interval does the Taylor series about $x = \frac{1}{5}$ converge? You may answer on the basis of the numerical results in (a), but for full credit you should give a mathematical justification for your answer.
3. Let $f(x) = 1/x$ and take $x_0 = 1, x_1 = 2, x_2 = 3, x_3 = 4$.
a) Find the Lagrange form, Newton form, and standard form of the interpolating polynomial $p_3(x)$. Check your answer in each case by verifying that $p_3(x)$ interpolates $f(x)$ at the given points.
b) Find an upper bound for the maximum error $E = \max_{1 \leq x \leq 4} |f(x) - p_3(x)|$.
4. Let x_0, x_1, x_2 be 3 distinct points and define $p(x) = L_0(x) + L_1(x) + L_2(x)$. Find the standard form of $p(x)$. Solve this problem two ways: (a) by direct computation using the definition of the Lagrange polynomials, (b) by applying the theorem which says there is a unique polynomial of degree $\leq n$ which interpolates a given function at $n + 1$ distinct points. (Hint for part (b): what function $f(x)$ does $p(x)$ interpolate?)
5. page 352, problem 14 (polynomial interpolation of physical data)
6. Let $f(x) = \sqrt{1-x^2}$ for $-1 \leq x \leq 1$. Using Matlab, plot the following approximations and comment on the results.
a) Taylor polynomials of degree $n = 0, 2, 4, 6$ about $x = 0$
b) piecewise linear interpolant with uniform mesh spacing $h = 2, 4, 8, 16$
c) piecewise linear interpolant using Chebyshev points with $n = 2, 4, 8, 16$