Math 471 Fall 2009 Homework 8 due: Mon Dec 14

chapter 5, polynomial approximation and interpolation

1. page 351, problem 9

Note: This problem asks you to derive an error bound for linear polynomial interpolation,

$$|f(x) - p_1(x)| \le \frac{1}{8} \max_{x_0 \le x \le x_1} |f''(x)| h^2 \ , \text{ where } h = x_1 - x_0.$$

You may prove this result by applying the theorem on the error in polynomial interpolation which was stated in class. The theorem says that given a function $f(x)$ and $n+1$ distinct points $a = x_0 < \cdots < b = x_n$, then $f(x) = p_n(x) + \frac{1}{(n+1)!} f^{(n+1)}(\zeta)(x - x_0) \cdots (x - x_n)$, where $p_n(x)$ is the polynomial of degree n that interpolates $f(x)$ at the points x_i and ζ is some point in the interval $[x_0, x_1]$.

2. Write a Matlab program to perform natural cubic spline interpolation at the uniform points on the interval $-1 \le x \le 1$ for $f(x) = |x|$. In Matlab use $f(x) =$ `abs(x)`. You may use the backslash command to solve the linear system for the spline coefficients. Let $s_n(x)$ denote the spline based on n intervals and investigate the convergence of $s_n(x)$ to $f(x)$ by running the program for different values of n. In the writeup, include the code and plots of $s_n(x)$ and the function $f(x)$ for $n = 2, 4, 6$. Answer the following questions.

Does $s(x)$ converge pointwise to $f(x)$ on $[-1, 1]$? Does $s(x)$ converge uniformly to $f(x)$ on $[-1, 1]$?

Note:

pointwise convergence means that $\lim_{n\to\infty} s(x) = f(x)$ for all $x \in [-1, 1]$

uniform convergence means that $\lim_{n\to\infty} \max_{x\in[-1,1]} |s(x) - f(x)| = 0$

Uniform convergence implies pointwise convergence, but the converse is false.

announcements

1. The online teaching evaluations are available from Friday Dec 4 to Tuesday Dec 15. Please complete the evaluations - they provide valuable feedback from students to instructors.

2. The final exam is on Wednesday, December 23, 10:30am-12:30pm, in 1084 East Hall. The exam will cover the entire course. A review sheet with sample problems will be distributed soon. You may use a non-programmable calculator to do arithmetic, but to receive full credit you must show all intermediate steps. You may use two sheets of notes (e.g. two sides of one page, i.e. a total of $187\,\text{in}^2 = 2 \times 8.5\,\text{in} \times 11\,\text{in}$). I will supply the exam booklets.