Math 557 Winter 2020 Homework 2 due: Thursday, February 13

1. Find the first three terms in the asymptotic expansion of $f(x) = \sqrt{1+x^2} - 1$.

(a) as $x \to 0$, (b) as $x \to \infty$

(c) Plot $f(x)$ and the 1-term, 2-term, and 3-term approximations. Make one plot for (a) with limits `axis([0 3 0 3])`, and another plot for (b) with limits `axis([0 5 0 5])`. Use a solid curve for $f(x)$ and a dashed curve for the approximations, and label each approximation with the number of terms.

2. Show that $\int_0^T e^{-xt^2} dt \sim \int_0^\infty e^{-xt^2} dt = \frac{1}{2}\left(\frac{\pi}{x}\right)^{1/2}$ as $x \to \infty$ for all $T > 0$.

Derive the result; don't just cite Watson's lemma.

3. In class we showed that $\int_0^\infty e^{-x\cosh t} dt \sim \left(\frac{\pi}{2x}\right)^{1/2} e^{-x}\left(1 - \frac{1}{8x}\right)$ as $x \to \infty$.

a) Find the next term in the asymptotic expansion.

b) Explain why the same result holds for $\int_0^T e^{-x\cosh t} dt$ for all $T > 0$.

4. Find the first three terms in the asymptotic expansion of $\int_0^\infty e^{-x\sinh t} dt$ as $x \to \infty$.

5. Let $f(x) = \int_0^T e^{xh(t)} g(t) dt$ and assume $h(0) = \max_{0\le t\le T} h(t), h'(0) = 0, h''(0) = 0, h'''(0) < 0$.

Note that an example of this was given on hw1, problem 5a.

Find the first term in the asymptotic expansion of $f(x)$ as $x \to \infty$.

6. In class we showed that $n! \sim \sqrt{2\pi} n^{n+\frac{1}{2}} e^{-n}$ as $n \to \infty$.

a) Find the next term in the asymptotic expansion.

b) Make a table with the following format.
column 1: $n = 1 : 10$
column 2: absolute error in 1-term asymptotic approximation
column 3: relative error in 1-term asymptotic approximation
column 4: absolute error in 2-term asymptotic approximation
column 5: relative error in 2-term asymptotic approximation
Summarize the results. Do they agree with what is expected from theory?

7. page 27 / 1 (iii) (integration by parts or term-by-term integration)

8. page 38 / 1 (i) , (iii) (Laplace's method)