Math 557 Winter 2020 Homework 5 due: Thursday, April 2

In the problems below find the first two terms in the asymptotic expansion of $w(z)$ as $z \to \infty$. These are 2nd order equations so there are two independent solutions; write them in the form $w(z) \sim e^{\lambda z} z^{\sigma} \left(\alpha_0 + \frac{\alpha_1}{z} + \cdots \right)$ as $z \to \infty$; you may take $\alpha_0 = 1$.

In the first two problems use the expansion $w(z) \sim e^{\lambda z} z^{\sigma} \left(\alpha_0 + \frac{\alpha_1}{z} + \cdots \right)$ as $z \to \infty$; this is method 1 in the class notes.

1. page 110 1(i) $w'' + \frac{1}{z} w = 0$

Hint for 1(i): note that (6.13) with $a_0 \neq 0$ is not satisfied; use the transformation following (6.20) in the form $z = t^2, w(z) = t^{1/2} u(t)$ to derive an equation for $u(t)$ for which (6.13) with $a_0 \neq 0$ is satisfied and then apply (6.14).

2. page 110 1(ii) $w'' + 2w' + \frac{2}{z} w = 0$

Besides finding the two asymptotic solutions, show that one of them is an exact solution.

Hint for 1(ii): eliminate the first derivative term and apply (6.14).

In the next two problems use the expansion $w(z) \sim \exp(\phi_0(z) + \phi_1(z) + \cdots)$ as $z \to \infty$; this is method 2 in the class notes.

3. page 110 2(i) $w'' - \frac{1}{z} w = 0$

4. page 110 2(ii) $w'' + z^2 w = 0$