Math 557 Winter 2020 Homework 6 due: Thursday, April 16

1. page 129 / 1(i) : $w'' + \lambda^2 x w = 0\,,\ \lambda \to \infty$

2. page 129 / 1(ii) : $w'' + (\lambda^2 x^2 + \lambda x + 1)w = 0\,,\ \lambda \to \infty$

In problems 1 and 2 we want to find asymptotic approximations to the solution; this can be done using the classical WKB method; however in this assignment use Murray's approach,

$$w(x,\lambda) \sim \exp(g_0(\lambda)\psi_0(x) + g_1(\lambda)\psi_1(x) + g_2(\lambda)\psi_2(x) + \cdots) \text{ as } \lambda \to \infty.$$

Include terms up to $g_2(\lambda)\psi_2(x)$; separate the argument of the exponential into real and imaginary parts; write the result in the clearest possible form.

3. page 130 / 5(i) : $w'' - \dfrac{1}{(1+\epsilon x)^2} w = 0\,,\ \epsilon \to 0\,,\ w(0) = a\,,\ w'(0) = b$

Problem 3 is an initial value problem with a small parameter $\epsilon \to 0$. Carry out the following steps, as we did for an example in class.

a) Find the solution in the form of a regular perturbation series,

$$w(x,\epsilon) \sim F_0(x) + \epsilon F_1(x) + \cdots \text{ as } \epsilon \to 0.$$

b) Find an approximation for $w(x,\epsilon)$ which is uniformly valid for $0 \le x \le O(\epsilon^{-1})$.

c) Show that the approximation in (b) reduces to the approximation in (a) for $x \to 0$.