Math 557 Winter 2020 Homework 7 due: Tuesday, April 28

method of matched asymptotic expansions

1. page 150 / 2(i) : $\epsilon u' + u = 1\,,\ u(0) = 0$

2. page 150 / 2(ii) : $\epsilon u'' - u = -1\,,\ u(-1) = u(1) = 0$

3. page 150 / 2(iii) : $\epsilon u'' + u' = a + bx\,,\ u(0) = 0\,,\ u(1) = 1$

4. Consider the nonlinear boundary value problem discussed in class,

$$\epsilon u'' + uu' - u = 0\,,\ u(0) = A\,,\ u(1) = B.$$

Find the uniform asymptotic approximation for the boundary conditions given below. Use the functions $f_\pm(s), g_\pm(s)$ defined in class. Sketch the solution in each case (by hand, MATLAB, or other software). Describe in words any special features of the solution, e.g. "The solution has a boundary/corner/internal layer at $x = \ldots$".

(a) $A = -1/2,\ B = -1$ (region 8)

(b) $A = 1,\ B = -1$ (region 9)

In problem (b) there are actually two solutions; this is possible because the problem is nonlinear; try to find both of them.

method of multiple scales

5. page 159 / 1 : $u'' + 2\epsilon u' + u = 0,\ u(0) = 0,\ u'(0) = 1$

Do the problem in the form stated above (this is slightly different than in the textbook, but the algebra is a bit simpler). Find the exact solution and the uniform asymptotic approximation, and plot them for $\epsilon = 0.5, 0.1$ and $0 \le t \le 10\pi$. The textbook says "find the first two terms" in the approximation; show that the second term $u_1(r,s)$ vanishes in this case, so the first term $u_0(r,s)$ is actually accurate to order $O(\epsilon^2)$.

Poincaré-Lindstedt method

6. Consider the initial value problem, $u'' + u = \epsilon u^3, u(0) = 1, u'(0) = 0, \epsilon > 0$, which describes a weakly nonlinear oscillation. In class we showed that the period is $T(\epsilon) = 2\pi(1 + \frac{3}{8}\epsilon + O(\epsilon^2))$ as $\epsilon \to 0$. Find the coefficient of the $O(\epsilon^2)$ term in $T(\epsilon)$.