Math 558   Applied Dynamical Systems   Winter 2023

hw3 , due: Thursday, March 9, 4pm

1. Consider the equation $F(a, x) = ae^x - x = 0$ with solution $(a_0, X_0) = (1/e, 1)$. We showed in class that $F_0 = F_{x0} = 0$, $F_{a0} \neq 0$, $F_{xx0} \neq 0$, and hence the bifurcation diagram in the (a, X)-plane has a turning point at $(a_0, X_0) = (1/e, 1)$. The purpose of this exercise is to derive this result using perturbation series.

Look for a solution near $(a_0, X_0) = (1/e, 1)$ in the form $X(a) = X_0 + X_1\epsilon + X_2\epsilon^2 + \cdots$,

a) ... where $\epsilon = a - a_0$.

b) ... where $a = a_0 + a_1\epsilon + a_2\epsilon^2 + \cdots$.

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2. Q2.5 the structural instability of a pitchfork bifurcation

3. Q2.8 the van der Waals equation of state and the cusp catastrophe

4. Q2.11 phase instability

5. Q2.9 Landau's theory of 2nd order phase transitions

Announcement

The midterm exam is on Wednesday, March 15 in class. It will cover everything up to and including the material covered in class on Wednesday March 8 (Chapter 1, Chapter 2, and part of Chapter 3). You may use one page (one side) of handwritten or typed notes, but do not photocopy the lecture notes.