Math 558 Applied Dynamical Systems Winter 2023

hw5 , due: Thursday, April 6, 4pm

1. Consider the equation $\dfrac{dx}{dt} = A(t)x$ with $A(t) = \begin{pmatrix} -1 + \frac{3}{2}\cos^2 t & 1 - \frac{3}{2}\sin t \cos t \\ -1 - \frac{3}{2}\sin t \cos t & -1 + \frac{3}{2}\sin^2 t \end{pmatrix}$.

a) Show that $A(t)$ is periodic with period $T = \pi$.

b) Show that the eigenvalues of $A(t)$ have negative real part for all t.

c) Show that $x(t) = \begin{pmatrix} -\cos t \\ \sin t \end{pmatrix} e^{t/2}$ is a solution.

Hence despite part b, the equation has a positive Floquet exponent.

Chapter 3, page 109

2. Q3.34 2-cycles of the Hénon map

3. Q3.44 fixed points of a complex map

Chapter 4, page 144

4. Q4.20 Lyapunov exponents for the Arnol'd cat map

Chapter 5, page 164

5. Q5.4 a rigid rotating body

6. Q5.11 finite-difference approximation of an ODE

Omit part (d), but in parts (a) and (b), determine whether the system is area-preserving.

Chapter 6, page 199

7. Q6.3 a saddle-node bifurcation