Math 571 Numerical Linear Algebra Winter 2021

hw4 , due: Thursday, March 18

1. The following Matlab code computes the QR factorization of a matrix A by Householder's method. Copy the code into an m-file and fill in the missing variables (denoted by $\cdots$). Submit the completed code. Print out the resulting Q, R using `format long`. Also print out the Q, R from Matlab's command `qr(A)` and comment on any differences between the results.

```
A = [1 1; -1 0; 0 1]; [m,n] = size(A);
for k = 1:···
    x = A(k:m,k);
    e = zeros(···,1); e(1) = 1;
    v = norm(x)*e - x; v = v/norm(v);
    for j = k:···
        A(k:···,j) = A(k:···,j) - 2*v*(v'*A(k:···,j));
    end
    H = eye(···) - 2*v*v';
    Q(:,:,k) = zeros(m,m); Q(1:···,1:···,k) = eye(···);
    Q(k:···,k:···,k) = H;
end
temp = eye(···); for k=1:···; temp = temp*Q(:,:,k); end
Q = temp; R = A;
```

2. Consider the overdetermined linear system: $x - y = 1,\ x + y = 0,\ x = 1$. Sketch the lines in the xy-plane. Find and plot the least squares solution.

3. The molecular weights of six nitric oxides (N_aO_b) were measured experimentally, yielding the results below. Using this data, perform a least squares fit to estimate the atomic weight of nitrogen and oxygen. You may use any method to solve the least squares problem.

NO (30.006), N_2O (44.013), NO_2 (46.006), N_2O_3 (76.012), N_2O_4 (92.011), N_2O_5 (108.010)

4. Prove the following statements.

a) $\kappa(A) \geq 1$ for any induced matrix norm

b) If U is unitary, then $\kappa_2(U) = 1,\ \kappa_2(UA) = \kappa_2(AU) = \kappa_2(A)$.

c) $\kappa_2(A) = \sigma_{\max}/\sigma_{\min}$ and if A is hermitian, then $\kappa_2(A) = |\lambda|_{\max}/|\lambda|_{\min}$

d) If $Ax = b$ and $(A + \delta A)(x + \delta x) = b$, then $\dfrac{||\delta x||/||x + \delta x||}{||\delta A||/||A||} \leq \kappa(A)$.

e) Consider the following example of $Ax = b,\ (A + \delta A)(x + \delta x) = b$.

$$\begin{pmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 32 \\ 23 \\ 33 \\ 31 \end{pmatrix}, \quad \begin{pmatrix} 10 & 7 & 8.1 & 7.2 \\ 7.08 & 5.04 & 6 & 5 \\ 8 & 5.98 & 9.89 & 9 \\ 6.99 & 4.99 & 9 & 9.98 \end{pmatrix} \begin{pmatrix} -81 \\ 137 \\ -34 \\ 22 \end{pmatrix} = \begin{pmatrix} 32 \\ 23 \\ 33 \\ 31 \end{pmatrix}$$

Verify that the two equations are correct. Compute $\dfrac{||\delta x||_\infty/||x + \delta x||_\infty}{||\delta A||_\infty/||A||_\infty}$, $\kappa_\infty(A)$ using Matlab.

5. Let A_h be the tridiagonal matrix on page 35 of the notes (including the factor $1/h^2$) associated with the difference equation $-D_+D_-u_i = f_i, i = 1 : N - 1, u_0 = u_N = 0$, where $h = 1/N$ is the mesh spacing. In class we found the e-values and e-vectors of A_h. Show that A_h is invertible and find a constant c independent of h such that $||A_h^{-1}||_2 \leq c$ for $0 < h \leq 1$.