Math 571   Numerical Linear Algebra   Winter 2021

Class Meeting Time: TuTh at 10-11:30am on Zoom (see the Canvas site for the link). Classes will be recorded and posted in Canvas for later viewing.

Instructor: Robert Krasny, 4830 East Hall, 763-3505, krasny@umich.edu

Office Hours: Tu 5-6:30pm, W 5-6:30pm, or email me for an appointment

Text: "Numerical Linear Algebra", by Lloyd N. Trefethen and David Bau, III (SIAM)

Math 571 is an introduction to numerical linear algebra. Three types of problems are considered, linear systems ($Ax = b$), eigenvalues and eigenvectors ($Ax = \lambda x$), and least squares problems ($\min ||Ax - b||_2$). We will study the accuracy, efficiency, and stability of numerical methods for the solution of these problems. As an application we'll consider finite-difference schemes for boundary value problems in 1D and 2D. Lecture notes will be posted in Canvas and at `http://www.math.lsa.umich.edu/~krasny/math571.html`.

Topics

1. vector/matrix norms, orthogonal matrices, projectors, singular value decomposition (SVD)

2. least squares problems, QR factorization, normal equations, Gram-Schmidt orthogonalization, Householder triangularization

3. stability, condition number, IEEE floating point arithmetic, backward error analysis

4. direct methods for $Ax = b$, Gaussian elimination, LU factorization, pivoting, Cholesky factorization

5. eigenvalues and eigenvectors, Schur factorization, reduction to Hessenberg and tridiagonal form, power method, inverse iteration, shifts, Rayleigh quotient iteration, QR algorithm

6. iterative methods for $Ax = b$, Krylov methods, Arnoldi iteration, GMRES, steepest descent, conjugate gradient method, preconditioning

7. applications: image compression by SVD, finite-difference scheme for two-point boundary value problem, Dirichlet problem for Laplace equation on a square, least squares data fitting

Recommended Prerequisites

a course in linear algebra (e.g. Math 214, 217, 417, 419)

a course in advanced calculus (e.g. Math 450, 451, 454)

some familiarity with Matlab or a similar tool, a Matlab tutorial is at the following site `https://www.mathworks.com/learn/tutorials/matlab-onramp.html`

Midterm Exam: Thursday, March 4, in class

Final Exam: Friday, April 30, 10:30am-12:30pm

Course Grade: homework = 30%, midterm exam = 30%, final exam = 40%

Class Policies

1. Homework will be assigned every 1-2 weeks and should be uploaded into Canvas by the assigned due date. Students are encouraged to study together and may talk to each other about the homework problems, but each student should write up and submit their own solutions without copying another student's solutions.

2. If you you need an accommodation for a disability, please let me know as soon as possible. In such cases a Verified Individualized Services and Accommodations (VISA) form must be provided at least two weeks before the need. The Services for Students with Disabilities (SSD) Office (G664 Haven Hall; http://ssd.umich.edu/) issues VISA forms.

3. Students are encouraged to ask questions in class, office hours, by email, and on Piazza.