Math 655 Topics in Fluid Dynamics - Hydrodynamic Stability Winter 2004

hw#2 , due: Tuesday, March 9

1. page 61 , problem 3.9 , Saffman-Taylor instability

2. page 107 , problem 6.2 , principle of exchange of stability

3. page 111 , problem 6.4 , horizontal velocity of a normal mode in thermal convection

4. page 114 , problem 6.10 , Lorenz model of thermal convection

5. page 116 , problem 6.12 , Swift-Hohenberg model of thermal convection

6. Consider thermal convection in two space dimensions (x, z) with free surface boundary conditions at $z = 0, 1$ and normal modes $\psi = ae^{st} \sin kx \sin 2\pi z, \theta = be^{st} \cos kx \sin 2\pi z$. (Note that the vertical variation here is different than the case studied in class.) Draw a streamline plot of the velocity field. Determine the linear growth rate s, the curve of marginal stability, and the critical Rayleigh number for these modes. Are these modes more stable or less stable than the ones proportional to $\sin \pi z$?

7. In the weakly nonlinear analysis of slightly supercritical thermal convection we encountered the operator L defined below, acting on domain D (see class notes of 2/3).

$$L = \begin{pmatrix} Pr\Delta^2 & -PrR_c\partial_x \\ -\partial_x & \Delta \end{pmatrix} \quad , \quad D = \left\{ u = (u_1 \, , u_2)^T \; : \; u_1 = u_{1zz} = u_2 = 0 \text{ on } z = 0 \, , 1 \right\}$$

The adjoint operator L^* is defined by the relation $< L^*u \, , v > = < u \, , Lv >$ for all $u, v \in D$, where $< u \, , v > = \int_0^1 \int_0^{2\pi/k} \left(u_1 v_1 + u_2 v_2 \right) dx \, dz$. Show that L^* is given by the expression below and that the given vector-valued function u is a solution of the homogeneous adjoint equation $L^* u = 0$.

$$L^* = \begin{pmatrix} Pr\Delta^2 & \partial_x \\ PrR_c\partial_x & \Delta \end{pmatrix} \quad , \quad u = \begin{pmatrix} \sin kx \sin \pi z \\ \dfrac{Pr(k^2 + \pi^2)^2}{k} \cos kx \sin \pi z \end{pmatrix}$$

8. In class it was stated that the marginal stability curve for double-diffusive convection is given by the expression below. Derive this result starting from the nondimensional linearized equations (see class notes of 2/5).

$$R = \frac{(k^2 + \pi^2)^3}{k^2} + \frac{S}{\tau}$$