MATH 671 Particle Methods in Scientific Computing Winter 2022

hw#1 due: Thursday, January 27 in class

0. (optional) Give a brief description of your academic background and research interests. If you work in a lab or research group, please give your supervisor's name and describe your project. One paragraph is fine.

1. Compute the DFT of the following vectors. Do this by hand (though you may check your answers using Matlab).

a) $v = (1, 0, 0, 0)^T$ b) $v = (1, 1, 1, 1)^T$ c) $v = (1, 0, 1, 0)^T$ d) $v = (1, -i, -1, i)^T$

2. In class we considered the vector $v = (\sin 2\pi k t_j)^T$, where $t_j = j/N$, $j = 0 : N-1$, with $N = 64, k = 16$. We saw that the DFT is zero everywhere except for two entries, $|\widehat{v}_{16}| = |\widehat{v}_{48}| = 4$. Now consider the case when k is not an integer, for example take $k = 16.5$. Plot the vector v and the absolute value of the entries of $\widehat{v}$; follow the format on page 3 of the lecture notes. How do the results compare with the case $k = 16$? What is similar, what is different? Can you explain the results?

3. The Danielson-Lanczos lemma for the case $M = 2$ says that $F_4 = \frac{1}{\sqrt{2}} B_4 (F_2 \oplus F_2) P_4$. Write out each factor explicitly and check the equality.

4. In class we considered the Danielson-Lanczos lemma for the case $N = 2M$.

a) Write down a version for the case $N = 3M$.

b) Let $N = 6$ and write down the matrix F_6. Note that in this case there are two factorizations, $6 = 2 \cdot 3$ (as in class) and $6 = 3 \cdot 2$ (as in part (a)). Write down both factorizations of F_6. In your answers, all matrix elements should be from the set $\{0, 1, \omega, \omega^2, \omega^3, \omega^4, \omega^5\}$, where $\omega = e^{-2\pi i/6}$ and $\omega^6 = 1$.

5. Express $n = 2022$ in binary form. Find the periodic shift n' and the bit reversed index n''. Give the answers in binary form and decimal form.

6. The discrete sine transform (DST) for $v \in \mathbb{C}^{N-1}$ is $\widehat{v}_n = \sqrt{\frac{2}{N}} \sum_{j=1}^{N-1} v_j \sin \frac{\pi n j}{N}$, for $n = 1 : N-1$.

Find the formula for the inverse DST. (hint : to get started consider the case $N = 2$)

7. Plot the balanced and unbalanced trigonometric interpolants of the following functions $v(x)$, for $0 \le x \le 1$, as in class. Take $N = 4, 8, 16, 32$.

a) $v(x) = 4x(1 - x)$

b) $v(x) = \sin \pi x$

c) $v(x) = \sin 2\pi x$

Discuss the results. Do the interpolants converge uniformly to $v(x)$ as N increases?