Math 555 — Fall 2016 — Homework Assignment 10 — Due Tuesday, November 29

(1) (Thursday?) Page 187, Problem 4(a,c). Use Rouché's Theorem and explain your reasoning.
(2) (Friday?) Page 188, Problem 14(b,d).
(3) (Saturday?) Page 188, Problem 16.
(4) (Sunday?) Use residue theory to find, for all real values of k, the Fourier transform
$$\hat{f}(k) := \int_{-\infty}^{\infty} f(x)e^{-ikx}\,dx$$
in the case that
$$f(x) := \frac{4x}{x^4 - 4x^3 + 8x^2 - 8x + 4}.$$
Hint: the denominator is a perfect square (and it is nonzero for all $x \in \mathbb{R}$).
(5) (Monday?) Page 188, Problem 17.
(6) (Tuesday?) Use residue theory to evaluate the integral
$$I := \int_{-1}^{1} \frac{\sqrt{1-x^2}}{x^4+16}\,dx,$$
where the positive square root is meant. Hint: let the function $R(z)$ be determined by the following properties: (i) it is analytic for $z \in \mathbb{C} \setminus [-1,1]$, (ii) $R(z)^2 = 1 - z^2$, and (iii) $R(z)$ is positive imaginary for $z > 1$ real. Evaluate the integral of $R(z)/(z^4+16)$ on the circle $|z| = 3/2$ and relate its value to I.
(7) (Wednesday?) Page 189, Problem 21.