Math 555 — Fall 2016 — Homework Assignment 11 — Due Thursday, December 8

(1) (Wednesday?) Page 190, Problem 23(b,d). Hint for part (b): Consider figure 39 and an integrand involving $[\log(z)]^2$ in place of $\log(z)$.

(2) (Thursday?) Let $P(x)$ be a polynomial of degree $2N$ for $N = 2, 3, 4, \ldots$ of the form

$$P(x) = -\prod_{n=1}^{N}(x - a_n)(x - b_n)$$

where $a_1 < b_1 < a_2 < b_2 < \cdots < a_N < b_N$ are the roots (all real). Show that $P(x) > 0$ on each interval $[a_n, b_n]$, and then evaluate the alternating sum:

$$\sum_{n=1}^{N}(-1)^n \int_{a_n}^{b_n} \frac{dx}{\sqrt{P(x)}}.$$

Hint: consider an appropriate analytic version of $\sqrt{P(z)}$ with a good choice of cuts, and try integrating its reciprocal over "dog-bone" shaped contours surrounding the cuts.

(3) (Friday?) Page 212, Problem 6.

(4) (Saturday?) Page 212, Problem 8.

Remark. "Solve the Dirichlet problem" means find a formula expressing the solution u in terms of given continuous boundary data, here $h(z)$ for $|z| = 1$. It turns out that to properly formulate the Dirichlet problem in an unbounded domain it is necessary to impose the additional condition that the solution u is a bounded function; otherwise the solution cannot be uniquely determined. Justify this latter statement. Hint: Consider $\ln(r)$.

(5) (*)(Sunday?) Page 212, Problem 9. [Neumann problem]

Remark/Hint. Parametrize the imaginary part of (14′), integrate by parts and use the Cauchy-Riemann equations. Important correction: this problem has a typo in it, because: (i) the solution to this problem only exists under an additional unstated condition on h, and (ii) when this condition holds, the solution is *not* unique. Justify these statements (i) and (ii). Hints: for (i) consider the Cauchy-Riemann equations, and for (ii) consider the function 1.

(6) (*) (Monday?) Find the unique solution to the Neumann problem for the upper half-plane under the condition that the solution u tends to zero as y goes to infinity. That is, determine $u(x, y)$ harmonic for $y > 0$ such that

$$\lim_{y \downarrow 0} \frac{\partial u}{\partial y}(x, y) = h(x),$$

where h is a suitable (explain what that means) continuous function on $\mathbb{R}$ that vanishes as $x \to \pm\infty$.

(7) (Tuesday?) Page 213, Problem 12.

(8) (Wednesday?) Page 213, Problem 14.

(9) (Thursday), Page 213, Problem 18