Math 555 — Fall 2016 — Homework Assignment 3 — Due Thursday, September 29

1. (Thursday?) Page 73, Problem 8.
2. (Friday?) Suppose a smooth curve C has two different parametrizations, say $z = z_1(t)$ for $a_1 \le t \le b_1$ and $z = z_2(t)$ for $a_2 \le t \le b_2$, with $z_j'(t) \neq 0$. Let f be continuous on C. Using the parametric form of the contour integral
$$\int_C f(z)\,dz = \int_a^b f(z(t))z'(t)\,dt$$
prove that the value of the integral is independent of the choice of parametrization.
3. (Saturday?) Page 73, Problem 10(b).
4. (Sunday?) Page 73, Problem 11.
5. (Monday?) Recall Green's Theorem:
$$\oint_C [P(x,y)\,dx + Q(x,y)\,dy] = \iint_R \left[\frac{\partial Q}{\partial x}(x,y) - \frac{\partial P}{\partial y}(x,y)\right] dx\,dy$$
for real functions P and Q with continuous first partial derivatives in the simply connected region R with positively-oriented boundary being a smooth closed Jordan curve C. Prove Green's Theorem in the special case that C has at most two intersection points with every horizontal and vertical line in the plane. *Hint:* in this case R can be represented as the region between two graphs
$$y = y_1(x) \quad \text{and} \quad y = y_2(x), \quad a \le x \le b, \quad \text{where} \quad y_1(x) \le y_2(x)$$
and also as the region between the two (sideways) graphs
$$x = x_1(y) \quad \text{and} \quad x = x_2(y), \quad c \le y \le b, \quad \text{where} \quad x_1(y) \le x_2(y).$$
Draw a good picture and use this information to evaluate the double integral using iterated integrals.
6. (Tuesday?) Page 74, Problem 15.
7. (Wednesday?) Page 75, Problem 20.