Math 555 — Fall 2016 — Homework Assignment 7 — Due Thursday, November 3

(1) (Thursday?) Consider the equation $Az\overline{z} + \overline{E}z + E\overline{z} + D = 0$ for a complex unknown z, where A and D are given real numbers, and where $E \in \mathbb{C}$ is a given complex number. Notice that the left-hand side is always real even if z is complex, so the equation describes one real condition on two real unknowns $x = \mathrm{Re}(z)$ and $y = \mathrm{Im}(z)$. Therefore we may expect this equation to describe one or more curves in the plane.

   (a) Show that this is the equation of a circle $(y - y_0)^2 + (x - x_0)^2 = r^2$ provided $A \neq 0$ and $E\overline{E} - AD > 0$. Express (x_0, y_0, r) in terms of (A, D, E).

   (b) Show that this is the equation of a line (either vertical $x = x_0$ or non-vertical in slope-intercept form $y = mx + b$) provided $A = 0$ and $E \neq 0$. Express x_0 in terms of and (A, D, E) for the vertical case and express (m, b) in terms of (A, D, E) for the non-vertical case.

(2) (Friday?) Consider the fractional linear mapping $w = w(z) = (az+b)/(cz+d)$.

   (a) What is the image of the mapping if the complex parameters (a, b, c, d) satisfy $ad - bc = 0$?

   (b) Suppose that $ad - bc \neq 0$. By solving for z in terms of w, show that the inverse mapping $z = z(w)$ is also a fractional linear mapping of the form $z(w) = (a'w + b')/(c'w + d')$ and that $a'd' - b'c' \neq 0$ holds.

   (c) Taking into account that a fractional linear mapping is unchanged if the four parameters are subjected to scaling by the same nonzero complex number (check it), show that the matrix of coefficients of the inverse mapping
$$\begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}$$
   may be expressed as the inverse matrix to the original coefficient matrix
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$
   whose determinant is $ad - bc$ nonzero by assumption.

   (d) Suppose that $w_1(z) = (az+b)/(cz+d)$ and $w_2(w_1) = (ew_1+f)/(gw_1+h)$ are two fractional linear mappings and that $ad-bc \neq 0$ and $eh-fg \neq 0$. Show that $w_2 = w_2(w_1(z))$ is again a fractional linear mapping, and show that its coefficients may be obtained from those of w_2 and w_1 by means of matrix multiplication.

(3) (Saturday?) Page 121, Problem 13.

(4) (Sunday?) Page 122, Problem 22(c).

(5) (Monday?) Page 135, Problem 6(b).

(6) (Tuesday?) Page 136, Problem 9(c).

(7) (Wednesday ?) Page 136, Problem 10(b). Hint: first work out what happens to $z^2 - 1$.