Mathematics and Technology: Past and Future

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- I thank the organizers for giving me the opportunity to give this talk.

Topics

- Part 1. Technology
- Part 2. Unreasonable Effectiveness of Mathematics
- Part 3. Bell Laboratories
- Part 4. Bell Labs Math Research Center
- Part 5. Quantum Computers

1. Technology

• This talk is about what mathematicians do in industry and applied areas.

• It is based on my experience at Bell Laboratories. At that time Bell Labs was the research and development part of American Telephone and Telegraph Corp, now AT&T.

• **Bell Labs**: "Worked on incremental improvements to the communications network, while simultaneously thinking far ahead, towards the most revolutionary inventions imaginable."

Technology-Smartphone-1

- The cellular smartphone is the most complicated technology product used by most people.

- It is a **two-way radio** and a **computer** ("microprocessor")

- The latest smartphone contain 2 billion transistors. Each transistor is roughly of size 20 nanometers= 2×10^{-8} meter.

- What has math got to do with it?

Technology-Smartphone-2

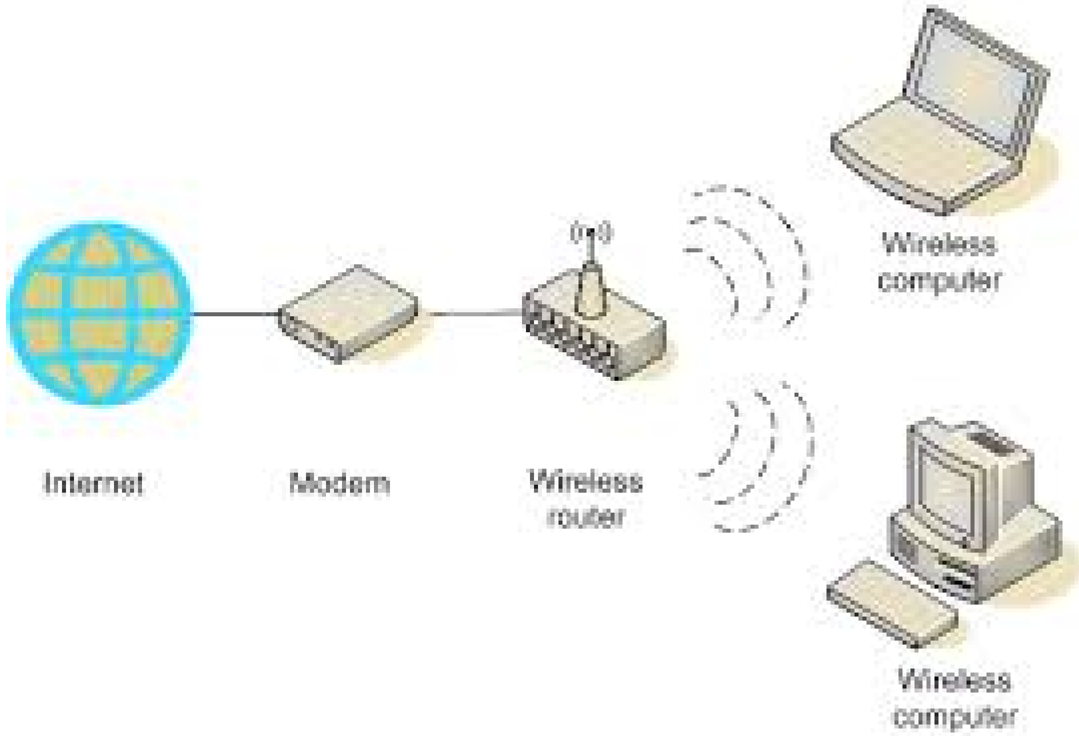
- Making a (sound or video) phone call.

- **Speech** is converted by cell phone microphone to electrical signal, then converted to a digital signal of 0's and 1's, then various digital mathematical operations done to it.

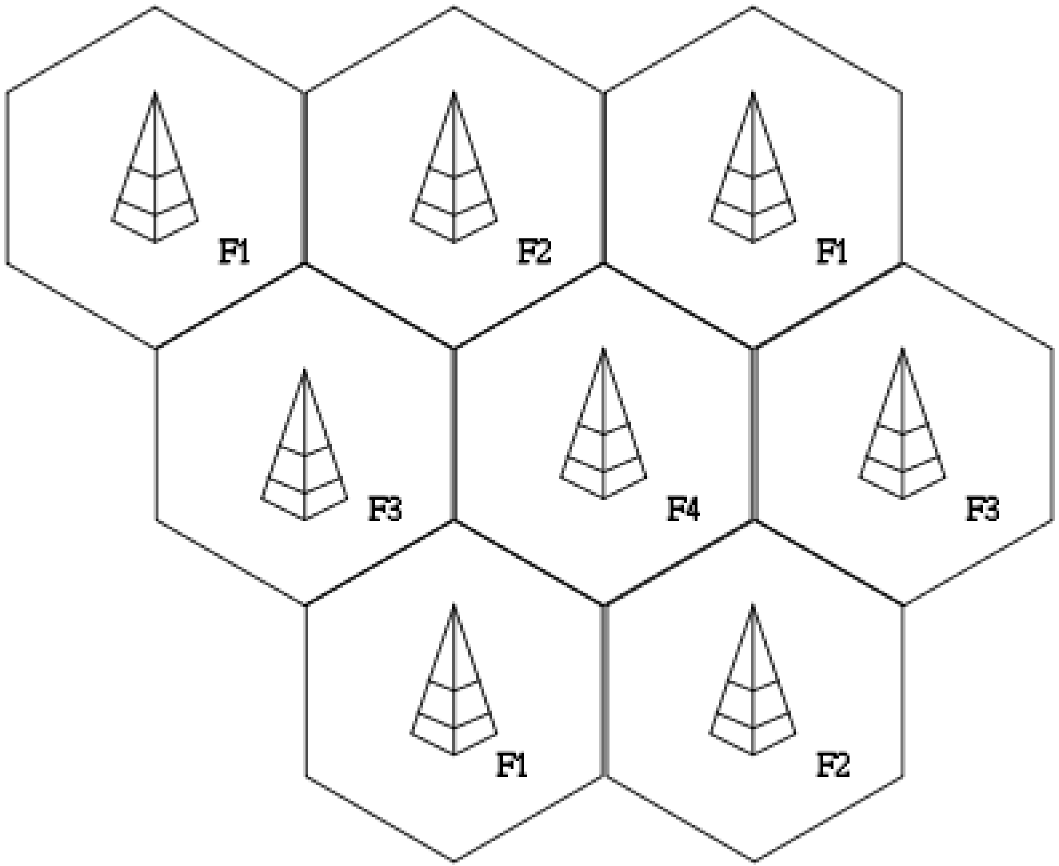
- Tasks to do: The signal is **sampled** and **quantized** into 0's and 1's. It is **compressed** to remove redundancy. The digital signal may be **encrypted** to ensure privacy. It is then **coded** for error correction.

- The data signal is sent by **modulated** radio waves (microwaves) to a **cellular base station** or **wireless router**. (**modulation** means: changing the shape of periodic waveform, the **carrier wave**, to transmit information)

Technology-Internet Access



Technology-Cellular Base Station Array



Technology-Smartphone-3

• Doing these mathematical operations quickly, and their containment in the hand-held device, and in base stations is made possible by **transistors** in the form of integrated circuits giving extremely fast switching, together with **optical fiber** technology for transmitting large amounts of data.

(An optical fiber acts as a **waveguide** for guiding electromagnetic waves in infra-red range. It carries **modulated** light signals at a precise frequency, powered by a laser.)

• **A Main Point:** A lot of mathematics is hidden in the process of making a phone call on a smartphone.

2. Unreasonable Effectiveness of Mathematics

• Paper: Eugene Wigner, The Unreasonable Effectiveness of Mathematics in the Natural Sciences (1960)

• "*The world is of baffling complexity and the most obvious fact about it is that we cannot predict the future.*"

• **Law of Nature**. A law that predicts future behavior. Mathematical in nature: quantitative and precise. It applies in limited circumstances. It claims that certain other things are **irrelevant** to the prediction.

• Picking out and discovering these laws, amidst the chaos of experience, is not obvious. The scientific enterprise aims to extend the range of these laws to wider domains.

Eugene Wigner Paper-2

• **Question.** Why are the laws of nature described accurately by mathematical equations?

• Newton's gravitational law near the earth applied by him to model the moon's motion. He calculated it with accuracy to within 4 percent.

• **But:** Now the gravitational law is repeatedly tested, now found to be accurate to one part in $1,000,000$. **Why is that?**

• **Wigner's Answer:** "The enormous usefulness of mathematics in the natural science is something bordering on the mysterious and there is no rational explanation for it."

Example: Electricity and Magnetism-1

- Hans Christian Oersted (1820) Discovered an electric current in a wire deflected a compass needle: **Electricity and Magnetism are related.**

- Electrical experiments of Englishman Michael Faraday (1791–1867), who described them, without much mathematics, using “lines of force”.

  Faraday cage (1836) method to shield a room from electromagnetic radiation.

History: Electricity and Magnetism-2

• James Clerk Maxwell, Scottish Physicist (1873) *A Treatise on Electricity and Magnetism*, He writes down equations for phenomena found by Faraday, with his own added experiments.

• 20 **Partial differential equations.** These describe forces acting *locally*, given by the effects of a (electromagnetic) field permeating space. [It was first **field theory**.]

• Equations predicted solutions: **Transverse electromagnetic waves**, of oscillating electric and magnetic fields, traveling at a constant speed.

• Electrical measurements showed this speed was close to the speed of light. Maxwell predicted: *visible light is such an electromagnetic wave.*

History: Electricity and Magnetism-3

• The consequences of Maxwell's equations were not all clear at the time that he wrote them. Nor was it clear in that it was the "right" theory.

• Consequences: Equations *predicted* new electrical and magnetic behaviors, in right circumstances.

[Figuring out what the equations implied took time! Experiments made to test the predictions.]

• Maxwell's equations are **unreasonably effective**.
"You get out more than you put in."

History: Electricity and Magnetism-4

• Heinrich Hertz (1857–1894) German physicist. PhD advisor Hermann Helmholtz said: "Please test Maxwell's theory of electromagetism." (Is light an electromagnetic wave?)

• In 1887, Hertz generated electromagnetic waves of length about a meter (**radio waves**). Built a *dipole antenna*, with a spark gap, set up standing waves between a pair of reflecting plates. He detected crests and troughs of a standing wave using a receiver spark gap.

• Hertz stated: "*It's of no use whatsoever [...] this is just an experiment that proves Maxwell was right. We just have these mysterious electromagnetic waves that we cannot see.*"

History: Electricity and Magnetism-5

• Oliver Heaviside (1850-1925) [“self-taught ” Electrician”] simplified Maxwell’s equations to 4 equations in 4 unknowns used today. Drew conclusions that followed from the equations.

• Heaviside used mathematical procedures that could not be rigorously justified at the time, but which gave good answers. (The **operational calculus**.)

• About his non-rigorous mathematics, he said:

“*I do not refuse my dinner simply because I do not understand the process of digestion.*”

• Mathematicians spent quite some time afterwards justifying these methods: for example **Norbert Wiener.**

Electromagnetic Spectrum-1

- Electromagnetic waves at various frequencies: oscillations per second measured in Hertz Hz

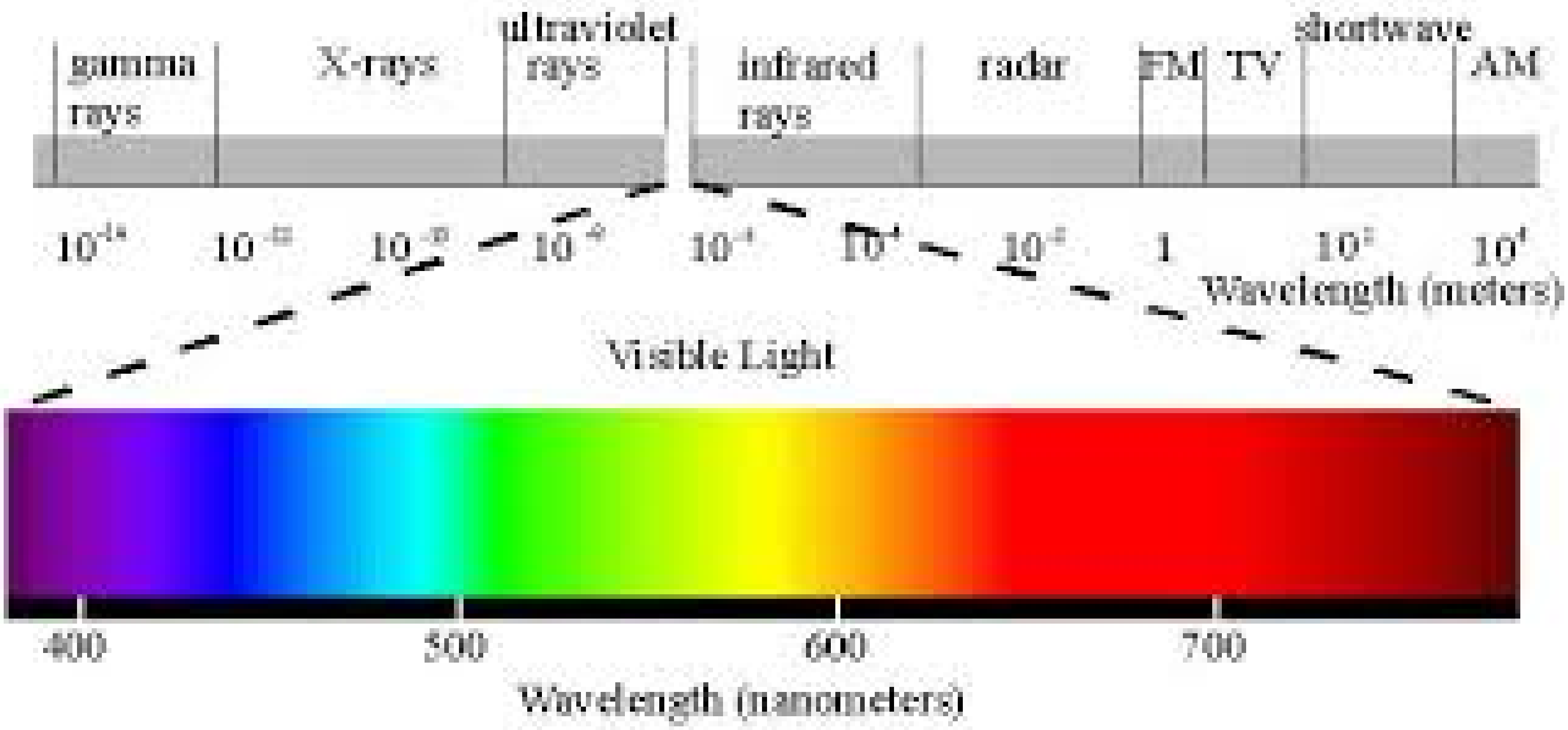
- **Visible light** 3×10^9 Hz, wavelengh 10^{-9} meter.

- **Infra-red** 10^8 Hz. Outside the visible range, less energetic (frequency used in optical fibers)

- **Microwaves** 10mm to 1 meter (frequency used in radar, cell phones).

- **Radio waves** 1meter to 100 meters

Electromagnetic Spectrum



Electromagnetic Spectrum-2

- Many cell phones work at an assigned microwave frequency 1850-1990 MHz (wavelength= 15 centimeters)

- Frequency of microwave oven, 2450 MHz (wavelength=12 centimeters)

- **Question.** Why don't you get cooked by your cell phone or laptop computer?

  Answer. It uses very low power.

Microwave Oven

Electromagnetic Spectrum-3 Microwave Oven

- Microwave oven has metallic walls and a mesh across the front entrance.

- It acts as a **Faraday cage** trapping the radiation inside.

- Recall: frequency of microwave oven, 2450 MHz (wavelength = 12 centimeters)

- The mesh is necessary to stop the microwaves, the holes in it are are significantly smaller than wavelength of the microwave radiation. (It does not stop visible light.)

Electromagnetic Spectrum-4 Microwave Oven

- Walls of oven reflect wave, sets up a **standing wave**. At certain places the waves reinforce, there is high energy. At other places, the wave cancels out, no heating.

- The rotating turntable at bottom of microwave oven is necessary! It moves the contents through the heating spots so they will be evenly heated.

- The original heating principle patented by Bell Laboratories, 1937. U. S. Patent 2,147,689.

3. Bell Telephone Laboratories

- Part of the telephone company: *American Telephone and Telegraph Corp.*. Served 90 percent of phones in USA. Called “Bell System” (after Alexander Graham Bell). combined production (Western Electric), research and distribution(Local phone companies). Regulated by U. S. government (Federal Communications Commission).

- *Mission.* To establish universal communication services.

- It has achieved that mission. In so doing, it has changed the world.

Bell Laboratories-2

- Provided steady funding to support Bell Laboratories.
- Built up a large stable group of dedicated researchers and developers.
- Budget: 10 percent research, 90 percent development.

Bell Labs- New York (till 1952)

Bell Labs- Murray Hill, NJ

Bell Laboratories-3

• Bell Labs **work culture** designed to encourage interaction across disciplines to break down communication barriers. To get engineers, physicists, materials scientists, mathematicians, statisticians, computer scientists, lab workers all to communicate to solve problems.

• People were **required** to address people on a **first name basis**, starting from the first day of work.

• Workers in all different disciplines had **offices mixed together**, doors kept open.

• Long, straight corridors, allowing random interactions through chance meetings in hall.

Bell Laboratories-4

- **Technical Support:**

Stock room had all kinds of electrical and mechanical equipment.

Machinists and technicians to build things.

Lab notebooks to record inventions, for patents.

Stock Room (A Stock Photo)

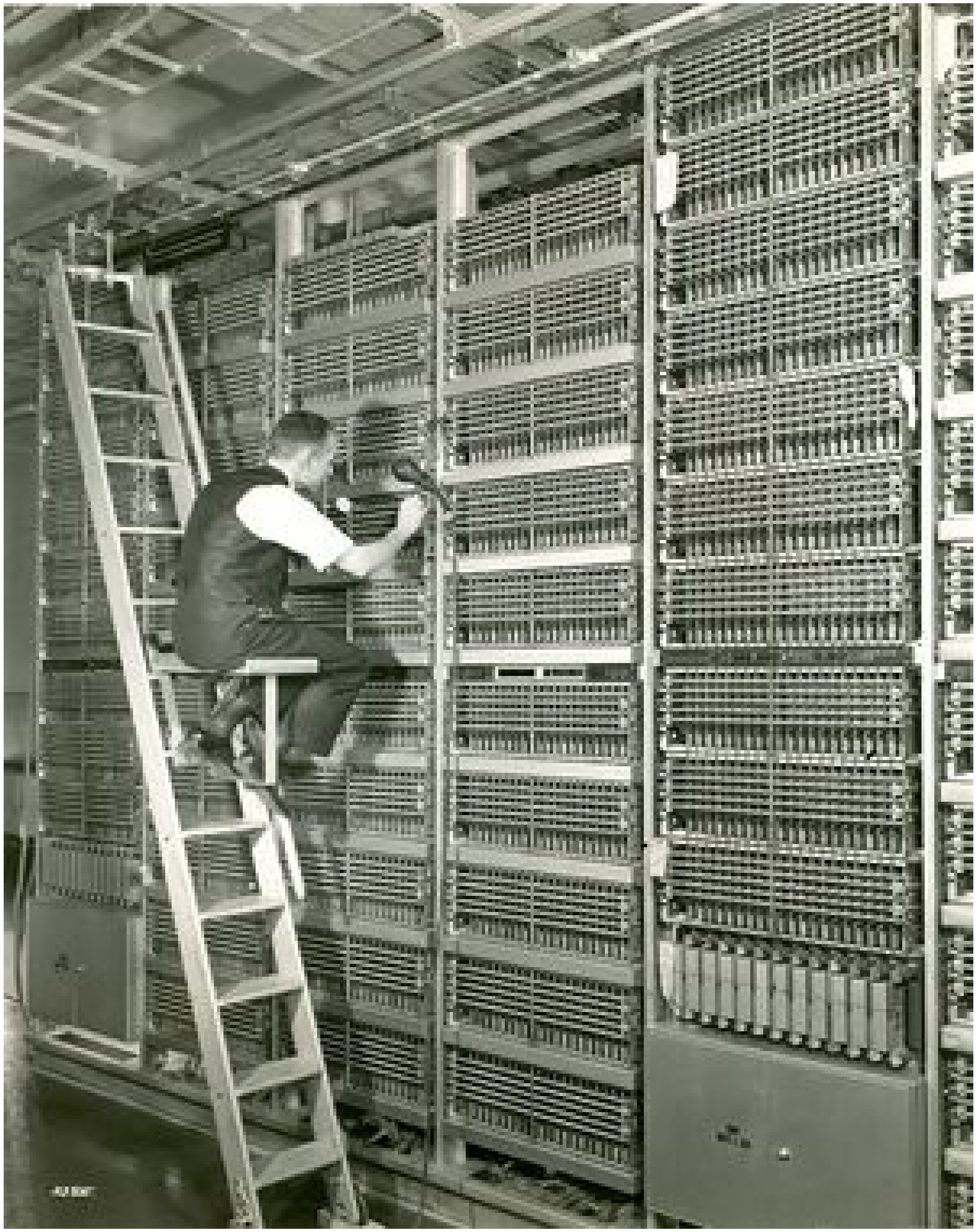
Bell Labs Mission-Oriented discoveries

• **Transistor.** (Bardeen, Brattain, Shockley (1947)) Sought as an *electronic switch* to replace mechanical telephone switches.

• **Solar Cell.** (Russell Ohl (1946)) practical version (1954) Sought to supply power for repeaters for phone lines in desert.

• **Laser.** (Schawlow, Townes (1960)). Acts as *optical amplifier* at optical wavelengths, used with optical fibers.

• **Cosmic microwave background radiation.** ("big bang") Observed by (Penzias and Wilson (1964)). Study done to determine microwave noise interference with AT & T satellite communication.



My history at Bell Labs

• 1974 Ph.D. in pure mathematics (Number theory), M. I. T.

• (1974-1980) Bell Labs Development Area (Operations research, Applied economics)

• (1980-2004) Math Research Center (Computational Complexity, Optimization, Wavelets, Quasicrystals)

• My math talent seems to be making connections across different fields.

• Finding connections between different fields is exciting. It can lead to productive discoveries, because it can enrich both fields with new ideas. An **old idea** in one field can instantly become a **new idea** in the other field, and vice versa.

4.Bell Labs Math Research Center

- **Math Research Center at Bell Labs.** To solve problems for the business and to develop new results in the mathematical areas related to the business. Goal oriented: but freedom for some people to work for years on long-term projects.

- **Theory role.**: Developing new mathematics to conceptualize general methods ("theory") and carry out processes ("algorithms").

- **Consulting role.** Helping solve problems in AT&T's business. May form part of a team assembled to solve such a problem.

- **Division of labor.** Math Center included people with different skills, from the most theoretical to those devoted to applications: Degrees in Math, Physics, EE, OR, CS, Statistics

Pure versus Applied Mathematics

• **Pure mathematics.** Investigates mathematical "internal structures" for their own sake. It finds new structures, new connections between structures, new methods. When are found it is often not clear what use they have.

• New pure mathematics "widgets" can go into the mathematical "stock room. "

• **Applied mathematics.** Makes use of the "widgets" to figure out what they are good for. Adjust and specialize the "widgets" to a more useful form for particular applications.

• These new "widgets" also go into the mathematical "stock room".

History: Bell Labs mathematics-1

• **Queueing Theory:** queues of calls in telephone routing systems [A. K. Erlang- Copenhagen telephone system, 1909]

• **Information Theory:** (Shannon (1948)): quantitative measurement of "information" as: "entropy" (amount of information needed to specify exact state of a system) Found theoretical limit on the **rate** of reliable transmission of information sent over a communications channel that makes errors. Showed theoretical existence of error-correcting codes to correct errors to achieve that limit.

History: Bell Labs mathematics-2

• **Error-correcting Codes:** (Shannon (1948)) These intelligently add **redundancy** to message strings, using extra "check bits." These allow detection and then correction of those errors in a received message.

• The last 70 years have been spent finding good codes, having fast encoding and decoding, to get close to the "**Shannon limit**". Good codes require large computer calculations to encode and decode. Codes approaching the limit were not practical until fast computing speeds achieved.

Space-Time Codes (1998) for 4G wireless use ideas from **non-commutative algebra**.

Consulting Role of the Math Center

Consulting work: Helping to formulate the problem. For an already formulated problem: supply an answer:

(1) There are mathematical methods that apply to the problem. Something is in the mathematical "stock room." (So: *Can form a team to work on a solution.*)

(2) No mathematical theory has been developed. Or, the problem is known to be too hard to solve efficiently. Nothing suitable in the "stock room." (So: *Change your problem or approach*)

- Both cases (1) and (2) can lead to **new mathematical research**, that can be either **pure** or **applied**.

Consulting: Fixing Cellular Base Stations-1

- **The Business Problem.** A change in regulation rules led to replacing an existing chip in a *cellular base station* with a new chip. The new chip had an incompatability with the other signal processing. It led to unacceptable rates of dropped cell phone calls. The problem showed up after delay and enormous numbers of base stations installed.

- Boxes built solidly, many screws, required returning them to factory to take apart and replace chip. Estimated cost to fix : $100 million US dollars.

Fixing Cellular Base Stations-2

- A math center member found a software solution to the problem. Reprogramming a plug-in chip on the outside of the box that could provide a fix to the signal problem.

- Avoided having to send the base stations back to the factory.

- Pleased A.T&T., and justified the salary of the entire Math Center for some years.

Karmarkar Algorithm-1

• Math Center member Narendra Karmarkar (1984) announces a new method for solving **linear programming** problems. This method said to be faster than the main method in use, the **simplex method.**

• Linear programming is used to optimize performance. (It finds solutions to a system of linear inequalities, to minimize a linear cost function.)

• Many commercial applications: Used by airline companies for scheduling planes, fuel loads and flight crews.

• Karmarkar's result was announced on front page of New York Times, November 19, 1984.

Interior-Point Methods—The Breakthrough

Breakthrough in Problem Solving

By JAMES GLEICK

A 28-year-old mathematician at A.T.&T. Bell Laboratories has made a startling theoretical breakthrough in the solving of systems of equations that often grow too vast and complex for the most powerful computers.

The discovery, which is to be formally published next month, is already circulating rapidly through the mathematical world. It has also set off a deluge of inquiries from brokerage houses, oil companies and airlines, industries with millions of dollars at stake in problems known as linear programming.

Faster Solutions Seen

These problems are fiendishly complicated systems, often with thousands of variables. They arise in a variety of commercial and government applications, ranging from allocating time on a communications satellite to routing millions of telephone calls over long distances, or whenever a limited, expensive resource must be spread most efficiently among competing users. And investment companies use them in creating portfolios with the best mix of stocks and bonds.

The Bell Labs mathematician, Dr. Narendra Karmarkar, has devised a radically new procedure that may speed the routine handling of such problems by businesses and Government agencies and also make it possible to tackle problems that are now far out of reach.

"This is a path-breaking result," said Dr. Ronald L. Graham, director of mathematical sciences for Bell Labs in Murray Hill, N.J. "Science has its moments of great progress, and this may well be one of them."

Because problems in linear programming can have billions or more possible answers, even high-speed computers cannot check every one. So computers must use a special procedure, an algorithm, to examine as few answers as possible before finding the best one — typically the one that minimizes cost or maximizes efficiency.

A procedure devised in 1947, the simplex method, is now used for such problems,

Continued on Page A19, Column 1

Karmarkar at Bell Labs: an equation to find a new way through the maze

Folding the Perfect Corner

A young Bell scientist makes a major math breakthrough

Every day 1,200 American Airlines jets crisscross the U.S., Mexico, Canada and the Caribbean, stopping in 110 cities and bearing over 80,000 passengers. More than 4,000 pilots, copilots, flight personnel, maintenance workers and baggage carriers are shuffled among the flights; a total of 3.6 million gal. of high-octane fuel is burned. Nuts, bolts, altimeters, landing gears and the like must be checked at each destination. And while performing these scheduling gymnastics, the company must keep a close eye on costs, projected revenue and profits.

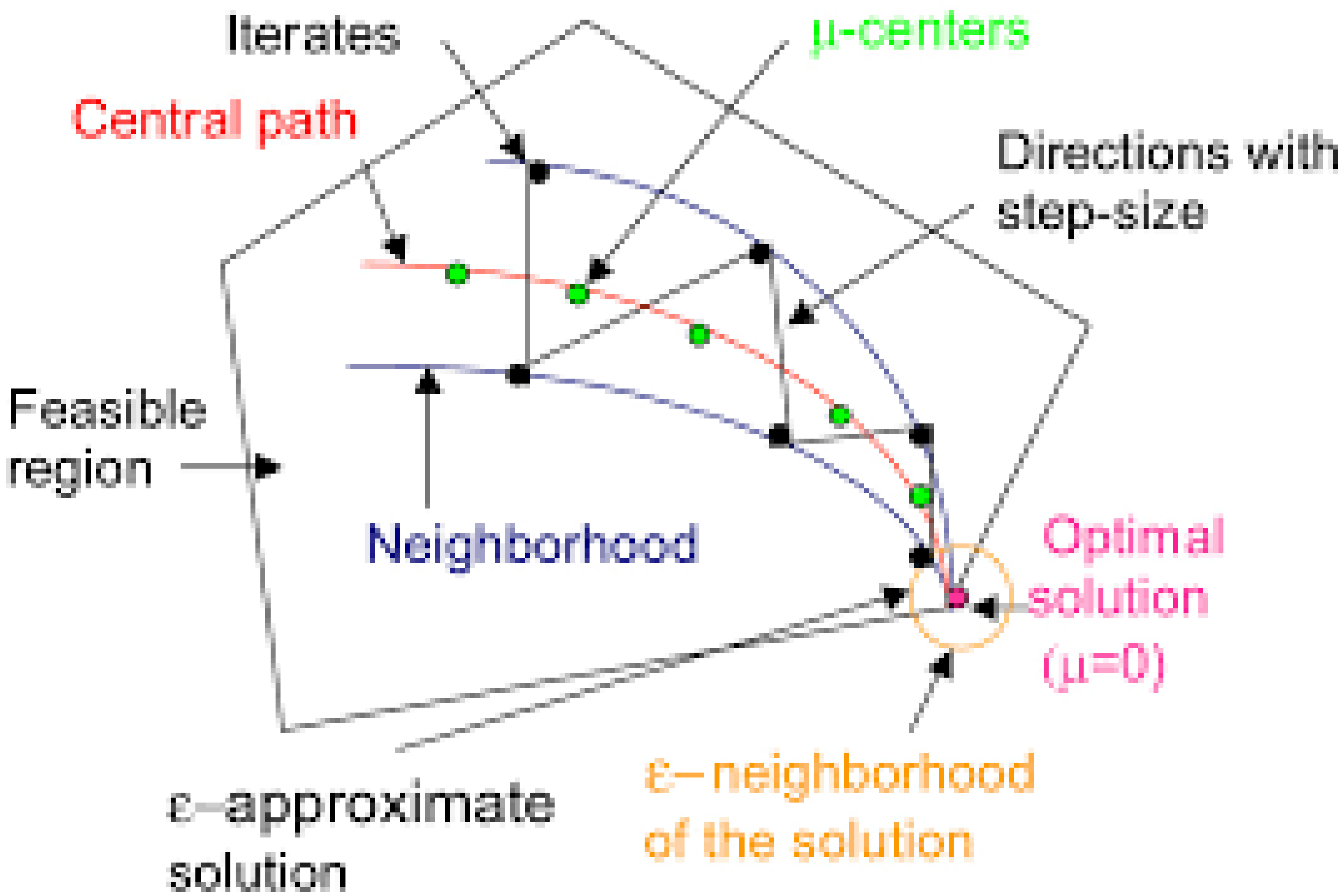
Like American Airlines, thousands of companies must routinely untangle the myriad variables that complicate the efficient distribution of their resources. Solving such monstrous problems requires the use of an abstruse branch of mathematics known as linear programming. It is the kind of math that has frustrated theoreticians for years, and even the fastest and most powerful computers have had great difficulty juggling the bits and pieces of data. Now Narendra Karmarkar, a 28-year-old Indian-born mathematician at Bell Laboratories in Murray Hill, N.J., after only a years' work has cracked the puzzle of linear programming by devising a new algorithm, a step-by-step mathematical formula. He has translated the procedure into a program that should allow computers to track a greater combination of tasks than ever before and in a fraction of the time.

Unlike most advances in theoretical mathematics, Karmarkar's work will have an immediate and major impact on the real world. "Breakthrough is one of the most abused words in science," says Ronald Graham, director of mathematical sciences at Bell Labs. "But this is one situation where it is truly appropriate."

Before the Karmarkar method, linear equations could be solved only in a cumbersome fashion, ironically known as the simplex method, devised by Mathematician George Dantzig in 1947. Problems are conceived of as giant geodesic domes with thousands of sides. Each corner of a facet on the dome

TIME MAGAZINE, December 3, 1984

Interior Point LP Method



Karmarkar Algorithm-2

- In 1986 A.T. & T. decided to **patent** the Karmarkar algorithm. In the 1980's United States patent rules were relaxed to allow patenting mathematical algorithms that have applications in technological processes.

- These patent applications caused controversy and were debated in the U. S. Senate.

- Patents 4,744,027 and 4,744,028 filed May 10, 1986.
U. S. patent granted May 10, 1988. Other versions granted in Canada, France, Germany. In Japan the patent application was challenged and went to Japan Supreme Court.

Karmarkar Algorithm-3

• 30 years later: **Interior point methods** are now routinely offered in linear programming computer packages, along with the **simplex method.**

• Both methods have survived. Each method works better than the other in some situations.

• **Interior point methods** have proved extremely useful in nonlinear problems. A huge amount of mathematical research developing theory is still going on in this area.

• **Pure Math**. The mathematics behind the Karmarkar algorithm has an incredibly interesting geometry. Its structure connects to pure mathematics. **algebraic geometry** and **integrable Hamiltonian dynamical systems**.

5. Quantum Computers

• Digital computers are being built ever faster and smaller, more transistors per chip (integrated circuit= IC). Chips are currently designed to minimize the disturbances quantum mechanics predicts will occur in very small regions. ("noise").

• **Question.** Can the laws of quantum mechanics be harnessed to make a faster computer?

• **Idea.** Don't fight the disturbances of quantum mechanics, but accept them and try to harness them for computations.

Quantum Computers-2

• Richard Feynman (1982) *Simulating physics by computers.* [Nobel Prize 1965].

Feynman: "*I think I can safely say that nobody understands quantum mechanics.*"

• The rules of quantum mechanics are "crazy" and are only understandable in mathematical terms. They are very different from classical behavior.

• **New Computational Resource. "Entanglement"** These are quantum correlations of behavior of many (properly prepared) separated particles.

Quantum Computers-3

It was shown at Bell Labs math center in the 1990's that there exist "hard" problems quickly solvable on quantum computer, not known to be quickly solvable on current computers.

- Peter Shor (1994), Fast factoring (and Discrete Logarithm) quickly solvable on a quantum computer

The algorithms of Peter Shor showed quantum computers are a threat to internet commerce because they can break encryption currently in use for Internet commerce.

Easy computational problem: (Multiplication) Given integers a, b compute their product $N = ab$.
Hard computational problem: (Factoring) Given a large integer N, find $a, b > 1$ with $N = ab$ (if one exists).

Quantum Computers-4

- **Question.** Can large-scale quantum computers be built?

  [Many groups are trying to do so, by many methods.]

- **Technical Difficulty.** Quantum computations that use entanglement are incredibly sensitive to external noise. "Collapse of the wavefunction" destroys a quantum computation.

- **Partial Solution.** (Peter Shor (1995)) showed that **Quantum Error Correcting Codes** exist. These codes can correct errors without directly measuring the quantum state that may have the error.

Quantum Computers-5

• As of March 2016, a working ion trap quantum computer with 5 ions was built that could factor $15 = 3 \times 5$ with Shor's method.

• This method is described as "scalable" so larger ones will be attempted. So the question is: *Will they work?*

• **Answer.** We won't know whether quantum computers scale up to large scale, to factor a number with 100 digits, for example, until one is built, and works!

• A quantum computer makes a new experimental test of the **laws of physics**. Will the laws remain **unreasonably effective** in this range? Or will new physics emerge?

Computers and Mathematics

- The increase in computing speed and power in recent years has happened so fast that there has not been time to figure out all the possible applications such computers have.

- There are wonderful opportunities ahead.

- Mathematical research is an experimental science. Computers are a wonderful tool is making such experiments. Proofs often come later, after convincing evidence is accumulated by computer.

The Last Slide...

- Besides describing applied mathematical work, I wished to honor **Bell Laboratories** for the wonderful environment it provided to me and many others for 30 years.

- Good work is made possible by good colleagues and good support.

- Thank you for your attention!

(References and Credits afterwards)

References and Credits-1

• Eugene Wigner, *The Unreasonable Effectiveness of Mathematics in the Physical Sciences,* Commun. Pure Applied Math. **13** (1960), no.1,

• Jon Gertner, *The Idea Factory: Bell Labs and the Great Age of American Innovation*, Penguin Press, London 2012.

• A. Michael Noll, *Memories: A Personal History of Bell Telephone Laboratories*, copyright 2015, A. Michael Noll.

• M. R. Garey and D. S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness,* W. R. Freeman and Company: San Francisco 1979

References and Credits-2

• S. J. Fortune, D. M. Gay, B. W. Kernighan, O. Landron, R. A. Valanzuela and M. H. Wright, *WISE Design of Interior Wireless Networks: Practical Computation and Optimization*, IEEE Computational Science and Engineering, Spring 1995, 58–68.

• Margaret H. Wright, *The Interior-point revolution in optimization: history, recent developments and lasting consequences,* Bull. Amer. Math. Soc. **42** (2004), No. 1, 39–56.

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