Problem # 1. Writing the augmented matrix of the system and reducing it, we get

$$\left[\begin{array}{cccc|c} 1 & 2 & 2 & 1 & 0 \\ 2 & 4 & 5 & 2 & 1 \\ 3 & 6 & 7 & 3 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 2 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{array}\right]$$
$$\longrightarrow \left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

Hence we get:

a) **Answer:** The reduced row-echelon form of the coefficient matrix is $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

b) **Answer:** The solutions are: y and w can be any numbers, $x = -2 - 2y - w$, and $z = 1$.

c) The augmented matrix of the new system reduces as follows

$$\begin{bmatrix} 1 & 2 & 2 & 1 & 1 \\ 2 & 4 & 5 & 2 & 1 \\ 3 & 6 & 7 & 3 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 2 & 1 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 2 & 1 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}.$$

Although we have not computed the reduced row-echelon form yet, we can stop now since it is clear that the reduced row-echelon form will have three leading 1s, so the rank of the augmented matrix of the new system is 3. **Answer:** The rank is 3.