Problem # 6.

a) We have

$$A^2 = A \Longrightarrow A^{-1}(A^2) = A^{-1}A \Longrightarrow (A^{-1}A)A = I \Longrightarrow A = I.$$

b) Let  $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ . Then A is not invertible but

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = A.$$

Answer: a) Yes, b) No.