Problem # 7.

a) Computing C^{-1}, we get

$$\left[\begin{array}{ccc|ccc} 1 & t & t & 1 & 0 & 0 \\ 0 & 1 & t & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc} 1 & t & 0 & 1 & 0 & -t \\ 0 & 1 & 0 & 0 & 1 & -t \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -t & t^2-t \\ 0 & 1 & 0 & 0 & 1 & -t \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right],$$

so $C^{-1} = \begin{bmatrix} 1 & -t & t^2-t \\ 0 & 1 & -t \\ 0 & 0 & 1 \end{bmatrix}$.

b) We have

$$(A-B)(A^{-1}-B^{-1}) = AA^{-1}-BA^{-1}-AB^{-1}+BB^{-1} = I-C^{-1}-C+I.$$

Substituting C and C^{-1} from Part a), we get

$$2I-(C+C^{-1}) = \begin{bmatrix} 0 & 0 & -t^2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Answer: a) $C^{-1} = \begin{bmatrix} 1 & -t & t^2-t \\ 0 & 1 & -t \\ 0 & 0 & 1 \end{bmatrix}$; b) $(A-B)(A^{-1}-B^{-1}) = \begin{bmatrix} 0 & 0 & -t^2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.