Problem # 4. The cosine of the angle between $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ can be computed as

$$\frac{(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})}{\|\vec{a}+\vec{b}\|\ \|\vec{a}-\vec{b}\|}.$$

We have

$$\vec{a}\cdot\vec{a}=\|\vec{a}\|^2=4, \quad \vec{b}\cdot\vec{b}=\|\vec{b}\|^2=9, \quad \text{and} \quad \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}=2.$$

¿From this,

$$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=\vec{a}\cdot\vec{a}-\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}-\vec{b}\cdot\vec{b}=\vec{a}\cdot\vec{a}-\vec{b}\cdot\vec{b}=4-9=-5,$$

$$\begin{aligned}\|\vec{a}+\vec{b}\| &=\sqrt{(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})}=\sqrt{\vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}}\\ &=\sqrt{4+2+2+9}=\sqrt{17}, \quad \text{and}\end{aligned}$$

$$\begin{aligned}\|\vec{a}-\vec{b}\| &=\sqrt{(\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})}=\sqrt{\vec{a}\cdot\vec{a}-\vec{a}\cdot\vec{b}-\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}}\\ &=\sqrt{4-2-2+9}=3.\end{aligned}$$

Therefore, the cosine of the angle between $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ is $-\dfrac{5}{3\sqrt{17}}$.

Answer: The cosine of the angle is $-\dfrac{5}{3\sqrt{17}}$.