2. Find a quadratic polynomial $f(t) = a + bt + ct^2$ such that $f(1) = 2$, $f(2) = -1$, and $f(3) = -8$.

Solution. We obtain the following system of equations in a, b, and c:

$$\begin{aligned} a + b + c &= 2 \\ a + 2b + 4c &= -1 \\ a + 3b + 9c &= -8. \end{aligned}$$

Solving the system, we get

$$\begin{array}{l} a + b + c = 2 \\ a + 2b + 4c = -1 \\ a + 3b + 9c = -8 \end{array} \longrightarrow \begin{array}{l} a + b + c = 2 \\ b + 3c = -3 \\ 2b + 8c = -10 \end{array} \longrightarrow \begin{array}{l} a + \quad -2c = 5 \\ b + 3c = -3 \\ 2c = -4 \end{array}$$

$$\begin{array}{l} a \quad - 2c = 5 \\ b + 3c = -3 \\ c = -2 \end{array} \longrightarrow \begin{array}{l} a \quad = 1 \\ b \quad = 3 \\ c = -2 \end{array},$$

so $a = 1$, $b = 3$, and $c = -2$.

Answer. $f(t) = 1 + 3t - 2t^2$.